A Course Responding to a UWM Study on Placement and Four Units of High School Mathematics

Wisconsin Mathematics Council
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Beth Ritsema
Mathematics Education Consultant
Grand Rapids, MI

Session Overview

- Highlights from a study by Professor Eric Key and Professor Richard O'Malley at UW-Milwaukee
- Thoughts on features of 4th units of high school math to better prepare students for post-secondary education
- How are some high schools and colleges addressing the issue of mathematical preparation?
- An overview of one high school 4th course as a model

The Effect of Four Years of High School Mathematics on Placement Testing at UWM
by Professor Eric Key and Professor Richard O'Malley[1]

Assume that we are considering a given pool of high schools students having completed 3 units of HS math. All of them have the same math ability as measured by some metric such as the ACT math score.

Assume that some of these students are given a 4th unit of high school mathematics (primarily precalculus) while the rest are given no additional math.

It is reasonable to anticipate that later when both groups are given a second standardized test that this group with 4 units of math will do "better" than the others with only 3 units of math.

The UW System Placement Examination in Mathematics is one such standardized test.

[1]http://www4.uwm.edu/Org/mmp/PDFs/fouryear%20copy3.pdf

Study Data and Analysis

New students at University of WI– Milwaukee

1515 students had 3 years of mathematics
2324 students had 4 years of mathematics

Primary analysis was focused on ACT ranges 16–19, 20–23, 24–27.

Placement test results were analyzed by subtest scores: Elementary Algebra (EALG), Intermediate Algebra (IALG), College Algebra (CALG), and Trigonometry.

ACT 16–19

467 students had 3 units and 475 students had 4 units
(25% of UWM's new freshmen fall in this range.)

- 5% of the students with 3 units of math scored well enough to be considered competent in algebra as measured by the IALG subscore

- 10% of the students with 4 units of math scored well enough to be considered competent in algebra as measured by the IALG subscore

Conclusion: For the students who scored 16–19 on the ACT, taking a 4th unit of high school mathematics (primarily precalculus) has not been effective.

ACT 20–23

“Neither group scores as well on EALG skills and concepts as their ACT score might lead us to hope.”

- 33.3% with 3 units scored lower than 500
- 22.8% with 4 units scored lower than 500

Conclusion: Students in this ACT range are probably not secure enough in their algebra skills to benefit from a tradition 4th unit of high school mathematics.

ACT 24–27

Data suggests that students who score in this ACT range do benefit from a 4th unit of math, that benefit seems to extend to solidifying previous topics and not learning more advanced mathematics.

Troubling: Roughly 33% of the students who have 4 units of high school mathematics still score below 500 on the IALG.

Recommendation: “A study should be undertaken to seek a different experience which would benefit these students and those in the 20—23 ACT range as well.” p. 9

Authors Conclusions and Recommendations

- In only some cases did a fourth year of high school mathematics study positively affect scores on the placement test.
- A discriminant in these cases was the level of a student's ACT math score. The researchers concluded that currently offered fourth-year high school mathematics courses (i.e., precalculus, advanced placement courses) were *not* beneficial for students whose ACT math score was less than 24.
- The researchers recommended "an alternative course of study in mathematics for the 4th unit of high school mathematics should be sought and developed for these students" (Abstract).

Key & O'Malley, 2014

Postsecondary Transition Issues

- Unacceptably high enrollments in non-credit bearing courses
- Three years of college preparatory mathematics is insufficient for college readiness (even 4 for some)
- Preparation for college mathematics is not necessarily preparation for calculus
- Much postsecondary learning involves quantitative skills and reasoning
- Preparing for independent learning
- Critical thinking

College Course-Taking Patterns
Two-Year Colleges

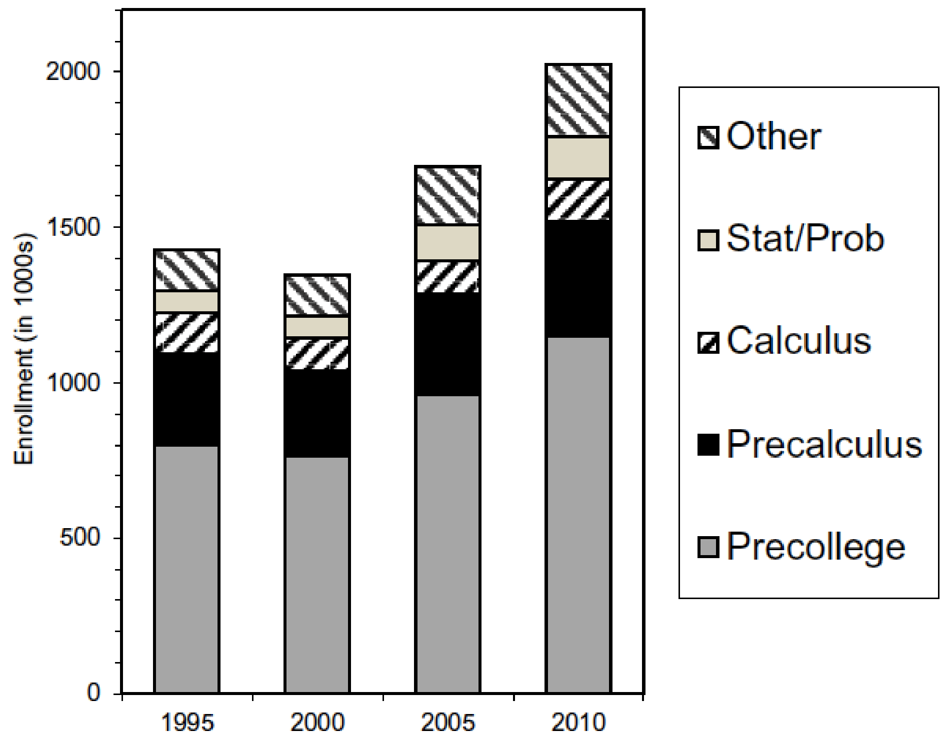


FIGURE TYE.4.1 Enrollment in 1000s (not including dual enrollments) in mathematics and statistics courses by type of course in mathematics programs at two-year colleges in fall 1995, 2000, 2005, and 2010.

Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States, Fall 2010, CBMS Survey,, p. 138
http://www.ams.org/profession/data/cbms-survey/cbms2010

College Board Survey of High School Class of 2010 - One Year Out

40% of students surveyed wished that had taken more/higher level math in high school.

Hart Research Associates, July 29–August 3, 2011

Wisconsin's Vision for Student Success in Mathematics

Educators and leaders at all levels must take on the challenge and the responsibility to ensure that all students are mathematically literate.

Students should analyze, reason, and communicate ideas effectively as they pose, formulate, interpret, and solve mathematical problems in a variety of situations to ensure success in a world beyond the classroom.

http://dpi.wi.gov/math

The Challenge

Designing a 4th unit of high school mathematics that best meets students needs and keeps them engaged.

What features might a 4th high school course include?

- Deepens and extends algebraic skills and conceptual understanding
- Interesting new relevant mathematics
- Helps students develop growth mindset (Carol Dweck)
- Allows for student choice (*Learning to Choose and Choosing to Learn*, Mike Anderson)
- Helps student become independent learners
- Expose students to additional technology for doing mathematics

Ideas Currently In Use in HS and College

- Courses based on adaptive computer software to practice and extend algebra and geometry skills
- Algebra II+ course
- Statway and Quantway from Carnegie Learning
- Modern Mathematics or Quantitative Reasoning
- Modern Mathematics or Quantitative Reasoning with an algebra review component

Quantitative Reasoning and Mathematical Decision Making

- Dana Center course to follow Texas AGA courses
- Materials designed for college general education courses
- Lessons from NCTM Illuminations and Core Math Tools and other online resources
- Individualized computer adaptive resources such as Dreambox, ALEX, MyMathLab
- *Transition to College Mathematics and Statistics*

Transition to College Mathematics and Statistics (TCMS)

TCMS is an alternative fourth-year mathematical sciences course designed to support college-readiness of non-STEM students. It is the product of four years of research, development, evaluation, and refinement funded by the National Science Foundation.

http://tcmsproject.org/

TCMS Design Features

- Mathematical strands developed in *coherent*, focused units that exploit *connections* to the other strands
- New mathematical ideas introduced in the context of *problem situations*
- Focus on *mathematical modeling*
- Emphasis on *student inquiry* and *sense-making*
- Full and strategic use of *technology* tools
- Designed to be used *following the third course* of an integrated mathematics program or with more conventional programs following an advanced algebra course (CCSS-M)
- Eight units allow *custom design of course* to meet various students needs

TCMS Unit Titles

Unit 1: Interpreting Categorical Data

Unit 2: Functions Modeling Change

Unit 3: Counting Methods

Unit 4: Mathematics of Financial Decision-Making

Unit 5: Binomial Distributions and Statistical Inference

Unit 6: Informatics

Unit 7: Spatial Visualization and Representations

Unit 8: Mathematics of Democratic Decision-Making

http://tcmsproject.org/

Published by McGraw-Hill Education

https://www.mheonline.com/program/view/2/16/ 2885/0076626261/

UNIT 2 Functions Modeling Change

Functions Modeling Change extends student understanding of linear, exponential, quadratic, power, circular, and logarithmic functions to model quantitative relationships and data patterns whose graphs are transformations of basic patterns.

Topics include linear, exponential, quadratic, power, circular, and base-10 logarithmic functions; mathematical modeling; translation, reflection, stretching, and compressing of graphs with connections to symbolic forms of corresponding function rules.

Sample Unit 2 Lesson 1 Applications Task

9 Accounting guidelines allow companies to recognize expense from wear and tear on equipment like a delivery van by reducing its value in business records each year. This accounting expense, called *depreciation*, reduces the company's taxable income and saves the company money.

a. If a company buys a \$40,000 delivery van and plans to depreciate its resale value to \$0.00 by a "straight line" method over 8 years, what is the annual depreciation allowance?

b. What function rule gives the depreciated value of the van on the company's books at any time n years after its purchase?

Sample Unit 2 Lesson 1 Connections Task

23 Use technology to make tables and graphs for the exponential function $f(x) = 10^x$ and the base 10 common logarithm function $g(x) = \log x$ on the same set of coordinate axes.

a. Based on an analysis of the graphs of the two functions, how are the graphs of $f(x) = 10^x$ and $g(x) = \log x$ related?

b. Based on an analysis of the two tables, how are the functions $f(x) = 10^x$ and $g(x) = \log x$ related?

c. How could your answer from Part b help you to draw a quick sketch of the base-2 logarithmic function $h(x) = \log_2 x$?

Sample Unit 2 Lesson 1 Algebra Review Tasks

41 Consider the function $y = \log x$.

a. Without using technology, decide if each statement is true or false. If false, explain why.

i. $\log 1{,}000 = 3$

ii. $\log(-1{,}000) = -3$

iii. $13 < \log 1{,}489 < 14$

iv. $-2 < \log 0.78 < -1$

b. Without using a calculator, create a table of values for the function $y = \log x$. Your table should contain at least five pairs of (x, y) values. Be prepared to explain how you decided on the x values to use in your table.

c. Sketch a graph of the function $y = \log x$.

d. Does the graph of $y = \log x$ have a vertical asymptote? If so, what is it? If not, explain why not.

e. Does the graph of $y = \log x$ have a horizontal asymptote? If so, what is it? If not, explain why not.

42 Solve each of the following equations.

a. $\frac{2x}{3} + \frac{x+3}{3} = 11$

b. $\frac{3}{x} + \frac{6}{x} = 15$

c. $\frac{x}{2} + \frac{x}{8} = 10$

d. $\frac{4}{x} + \frac{5}{4x} = 8$

Sample Unit 4 Lesson 1 Algebra Review Tasks

47 Rewrite each expression in an equivalent form that does not contain parentheses.

a. $3(2x + 6) - 10$

b. $0.03(2x^2 + 25x)$

c. $5(30(1.05^x) + 300)$

d. $(2x + 1)^2 - 5(2x + 1) + 15$

e. $\left(\sqrt{4x + 8}\right)^2$ for $x \geq -2$

48 Find the exact value of each expression if $x = -3$ and $y = 4$.

a. x^2

b. $-y^2$

c. x^{y+1}

d. y^x

e. $(x - y)^{-2}$

f. $\left(\frac{x}{y}\right)^y$

UNIT 4

Mathematics of Financial Decision-Making

Mathematics of Financial Decision-Making extends student facility with the use of linear, exponential, and logarithmic functions, expressions, and equations in representing and reasoning about quantitative relationships, especially those involving financial mathematical models.

Topics include forms of investment, simple and compound interest, future value of an increasing annuity, comparing investment options, continuous compounding and natural logarithms; amortization of loans and mortgages, present value of a decreasing annuity, and comparing auto loan and lease options.

Lesson 1	Financial Decision-Making: Saving
Lesson 2	Financial Decision-Making: Borrowing
Lesson 3	Looking Back

13 New product sales often follow predictable patterns. Sales of a product can start off slowly and then pick up as more people learn about it, and finally level off once the market is saturated.

The **logistic function** $S = \dfrac{1}{k + ae^{-bt}}$

is often used to model growth situations like that above. In this model, S is number of sales (in millions), t is elapsed time (in weeks), and parameters a, b, and k are positive numbers.

a. Graph the logistic function for $k = 0.2$, $a = 3$, and $b = 0.25$. Then draw a rough sketch of the graph.

  i. Does the pattern of change match the described pattern of sales? Explain.

  ii. Estimate from the graph when 1 million sales can be expected. Estimate the total sales expected for the product over time.

b. If release of a new product has been highly anticipated, sales of the product can start off rapidly and then level off. Draw a sketch illustrating this pattern of change in sales.

c. The mathematical models in Parts a and b have many other applications. Match the following phenomena with one or both of the above models.

  i. Learning of a new mathematical topic

  ii. Population growth

  iii. Spread of epidemics

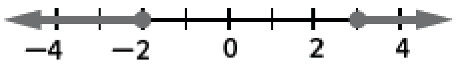
College Readiness Assessments

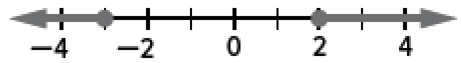
In addition to continuing use of important competencies in the student text, you will find College Readiness Assessment (CRA) sets in the unit resource masters for each lesson in the course. The exercises will help students build skills in strategic areas. The exercises are in multiple-choice format as is commonly found on mathematics placement tests.

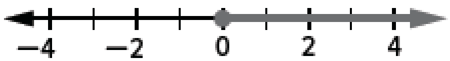
Editable and pdf formats

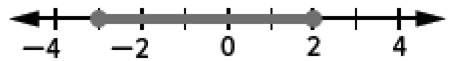
CRA Sample Items Set 1

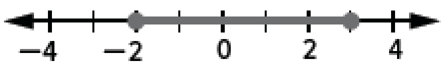
4. Which graph represents the solution to $(x-3)(x+2) \geq 0$?

(a) 

(b) 

(c) 

(d) 

(e) 

5. Simplify $\left(\frac{x^3}{3z}\right)^{-3}$.

(a) $\frac{-9z^3}{x^9}$ **(b)** $\frac{-27z^3}{x^9}$ **(c)** $\frac{9z^3}{x^6}$ **(d)** $\frac{-9z^3}{x^6}$ **(e)** $\frac{27z^3}{x^9}$

6. How many x-intercepts does the graph of $f(x) = x(x^2-9)(x^2+10)$ have?

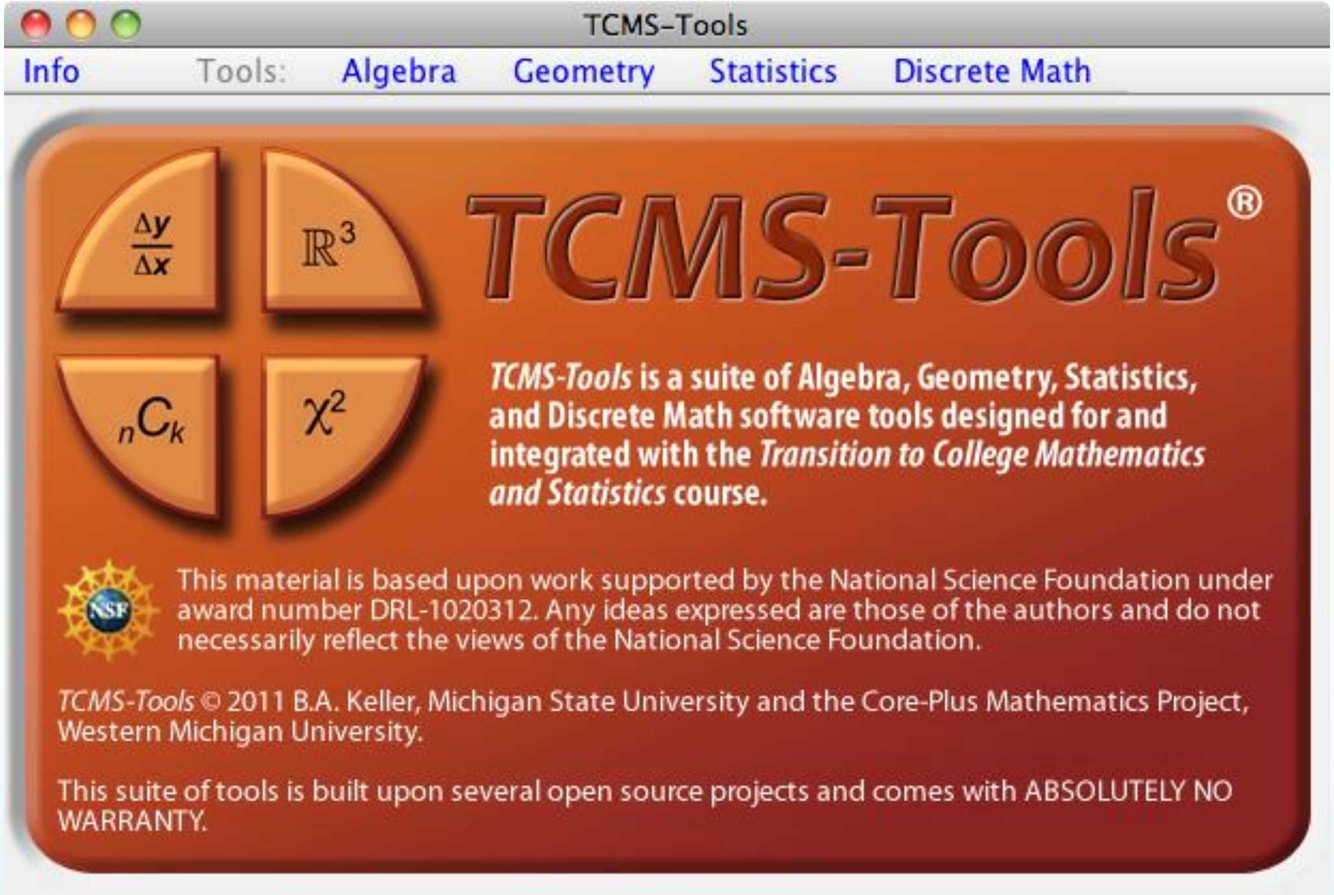
(a) 5 **(b)** 4 **(c)** 3 **(d)** 2 **(e)** 1

7. How many solutions does this system of equations have?

$$\begin{cases} 2x - y = 6 \\ -6x + 3y + 18 = 0 \end{cases}$$

(a) 1 **(b)** 2 **(c)** 3 **(d)** infinitely many **(e)** none

Public domain software developed with TCMS



Available at: http://tcmsproject.org/TCMS-Tools.html

UNIT 6 Informatics

Informatics develops student understanding of the mathematical concepts and methods related to information processing, particularly on the Internet, focusing on the key issues of access, security, accuracy, and efficiency.

Topics include elementary set theory and logic; modular arithmetic and number theory; secret codes, symmetric-key and public-key cryptosystems; error-detecting codes (including ZIP, UPC, and ISBN) and error-correcting codes (including Hamming distance); and trees and Huffman coding.

Modular Arithmetic – Launch

Download Clip 1 from:
http://www.wmich.edu/tcms/classroomvideo.html

Modular Arithmetic – Investigation

Download Clip 3 from:
http://www.wmich.edu/tcms/classroomvideo.html

Voices of TCMS Teachers

What do you see as the major strengths of TCMS?

- It reaches out to a select population of students that we previously had nothing to offer them.

- TCMS is the perfect class for collaborative learning. Students learn to actually read in a math class, they learn how to make mistakes and learn from them as opposed to being discouraged by them, and they also get a deeper understanding of the mathematical material since the topics are all in a real-world context.

- Students become independent learners which will serve them well in postsecondary courses and life.

More Voices of TCMS Teachers

- This course really made the teacher and students think about the mathematics being taught and learned. It gave a lot of students who were unsuccessful in Algebra 2 an opportunity to be successful. They enjoyed most topics and the contexts were very engaging. Many of my students left at the end with a view of mathematics as being useful.

- One of the strengths is the connection that TCMS makes to the professional careers that exist today. Students can see the relevance in learning the mathematics, even if it's not [always] the field of study they are interested in.

Voices of TCMS Students

Would you recommend this course to junior students considering a math course to take for next year?

- Yes, because it seems like we learn things that are much more applicable to daily life than other math classes. My favorite was creating codes.

- I would, because you keep reviewing algebra that will help in college and will learn new interesting statistics that should be helpful in life or work. But if a student is looking for a really easy senior math course, maybe not this one…

- Yes, because you learn a lot of math and get better prepared to learn more independently in college where you do not get as much help from teachers.

Voices of TCMS Students

What topics did you find most interesting? Least interesting or most difficult?

- This course takes skills and ideas learned from previous courses, reviews them, and expands upon the concepts that may have been misunderstood previously. Most interesting: Different voting methods gave different outcomes. Most Difficult: Some of the graphing we were required to do, involving transformations especially.
- I like learning about statistical significance. I better understand how to tell if something is very risky or not so risky. I learned a lot in the unit about credit cards, loans, and savings, but some of it like the logarithms was hard for me.

Voice of a College Student Enrolled in a Psychology Course

Dear Mr. Hummel,

Hi, how are you? I hope your year is going well! I just wanted to share with you that because of you, I know a thing or two in college!

I did not take a math class this semester, but I found something interesting in one of my other classes.

In my psychology class, I am currently reading about RANDOM SAMPLING! And POPULATION! And CORRELATION COEFFICIENTS! And STATISTICAL SIGNIFICANCE! Math is everywhere. :)