

WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET
WISCONSIN MATHEMATICS COUNCIL
March 2 – 6, 2015

Problem Set #1

Score: _____
(For Scorer's Use Only)

Name: _____

Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

For this first problem set, calculators are not allowed. They may be used for the remainder of the meet only, starting with Problem Set #2.

Answers

1. (1 point)

A certain gold bar weights nine-tenths of a kilogram plus nine-tenths of itself. How much does the gold bar weigh?

_____ kg

2. (3 points)

The complex conjugate of the number z is \bar{z} . For how many complex numbers z is $z^2 = \bar{z}$?

3. (5 points)

If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ for $-1 < x < 1$, then rewrite

$f\left(\frac{3x+x^3}{1+3x^2}\right)$ in terms of $f(x)$.

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Problem Set #2

Score:

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Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

Nine black socks and six brown socks are in a drawer.
Two socks are removed at random from the drawer.
What is the probability that they have the same color?
Write your answer as a reduced fraction.

2. (3 points)

How many points with integer coordinates are in the interior of the circle centered at (20,15) with radius 3?
Note: do not include points on the circle itself.

3. (5 points)

Let A and B be digits from the set $\{0, 1, 2, \dots, 9\}$. Let r be the two-digit integer AB and let s be the two-digit integer BA, so that r and s are members of the set $\{00, 01, 02, \dots, 99\}$. Compute the number of ordered pairs (A, B) such that $|r - s| = k^2$ for some integer k .

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Problem Set #3

Score:

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Name: _____

Team: _____

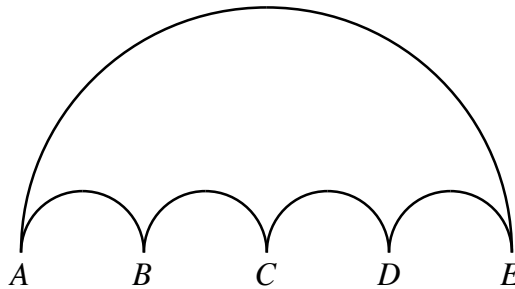
[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

$A, B, C, D,$ and E are collinear with $AB = BC = CD = DE = 1$.

All five shapes are semicircles. What is the perimeter of the figure?



2. (3 points)

Write all the ordered pairs of numbers (x, y) that satisfy both $x(x + y) = 9$ and $y(y + x) = 16$.

3. (5 points)

Using the fact that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$,

evaluate the sum $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$.

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Problem Set #4

Score: _____
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Name: _____

Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

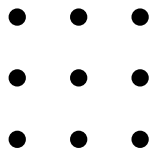
Answers

1. (1 point)

Let $a = 2^{1000}$, $b = 3^{600}$, and $c = 10^{300}$. Arrange the letters a , b , and c in increasing order. For example, if $a < b < c$, write it this way in the answer blank.

2. (3 points)

How many circles in the plane pass through at least three of the nine points $(0, 0)$, $(0, 1)$, $(0, 2)$, $(1, 0)$, $(1, 1)$, $(1, 2)$, $(2, 0)$, $(2, 1)$, and $(2, 2)$?



3. (5 points)

Find the real numbers $a \leq b \leq c \leq d \leq e$ with $a, b, c, d, e \in [-2, 2]$, such that $a + b + c + d + e = 0$, $a^3 + b^3 + c^3 + d^3 + e^3 = 0$, and $a^5 + b^5 + c^5 + d^5 + e^5 = 10$.

$a =$ _____, $b =$ _____,

$c =$ _____, $d =$ _____,

$e =$ _____

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Team Problem Set (Page 1)

Score:
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Team: _____

Captain: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (10 points)

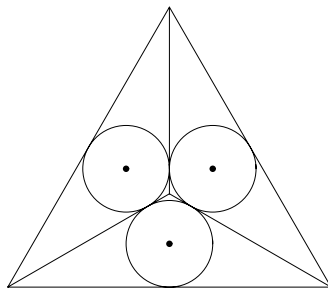
Let x , y , and z be consecutive integers such that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} > \frac{1}{2015}.$$

Find the maximum value of $x + y + z$.

2. (10 points)

The large triangle below is an equilateral triangle with side length 1. Three circles are inscribed in the smaller triangular sections. What is the area of one of those circles?



3. (10 points)

If the sum of the first n terms of a sequence is $n^2 + 5n$, what is the 2015th term of the sequence?

Team Problem Set (Page 2)

4. (10 points)

The city council has nine members, who serve on different committees. Each committee has three members, and no two members serve together on more than one committee. Determine the largest possible number of committees the council can have. _____

5. (10 points)

Eight students were given a basket with 32 apples to divide among themselves. Mary got one apple, Deb got two apples, Wendy got three apples, and Jennifer got four apples. Derek Smith took as many apples as his sister, Joe Brown took twice as many as his sister, Chris Jones took three times as many as his sister, and Scott Robins took four times as many as his sister. Give the last names of the girls. Mary _____

Deb _____

Wendy _____

Jennifer _____

6. (10 points)

Two integers are considered relatively prime if their greatest common factor (GCF) is 1. How many positive integers less than 2015 are relatively prime to 2015? _____