WISCONSIN MIDDLE SCHOOL STATE MATHEMATICS MEET
WISCONSIN MATHEMATICS COUNCIL
March 2 – 6, 2015

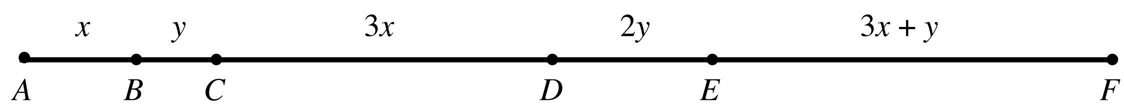
Solutions

Problem set #1

1. Let C = # of correct answers and I = # of incorrect answers. We set up a system of equations and solve for C:

$$\begin{aligned} C + I &= 12 \quad \rightarrow \quad 3C + 3I = 36 \\ 5C - 3I &= 20 \qquad\quad\ \ \underline{5C - 3I = 20} \\ &\qquad\qquad\quad\ \ 8C \qquad = 56 \quad \rightarrow \quad C = \boxed{7} \end{aligned}$$

2. For the entire year, Mr. Halvorson wants to have a yearly earnings of $\$4{,}000 \times 12 = \$48{,}000$. For the first 7 months, he will earn $\$4{,}600 \times 7 = \$32{,}200$. This leaves \$15,800 left to earn for the remainder of the year. Since he is taking December off, he only has 4 months left to earn this much money. This gives an average of $\$15{,}800 \div 4 = \boxed{\mathbf{\$3950}}$ per month.

3. 

Let $AB = x$ and $BC = y$. Then $AD = x + y + 3x = 4x + y = 33$. $BF = y + 3x + 2y + (3x + y) = 6x + 4y = 62$. From the AD equation, $y = 33 - 4x$, and substituting into the BF equation, we have $6x + 4(33 - 4x) = 62 \rightarrow 6x + 132 - 16x = 62 \rightarrow 70 = 10x \rightarrow x = 7$. Plug this into the AD equation to get $y = 33 - 4(7) = 5$. Therefore, $EF = 3(7) + 5 = \boxed{\mathbf{26.}}$

Problem set #2

1. The sum of A, B, and C ends in B, so A + C = 10. A is also the tens digit of the sum, so A is 1, which makes C = $\boxed{\mathbf{9.}}$

2. Let x, y, and z be the width, length, and height, respectively. Then $xy = 120$, $xz = 96$, and $yz = 80$. If you make this ratio, $\frac{yz}{xy} = \frac{z}{x} = \frac{80}{120} = \frac{2}{3}$ (this eliminates y from the equations, reducing it down to 2 variables), and solving for z gives us $z^2 = (xz)\left(\frac{z}{x}\right) = (96)\left(\frac{2}{3}\right) = 64$.

Therefore, the height of the box $z = \boxed{\mathbf{8.}}$

3. There are 8 non-winner bottles and 2 winner bottles. We write the probabilities of selecting the desired bottles according to the problem and multiply them together. We need three non-winners, followed by a winner: $\frac{\cancel{8}}{10} \cdot \frac{\cancel{7}}{9} \cdot \frac{6}{\cancel{8}} \cdot \frac{2}{\cancel{7}} = \frac{12}{90} = \boxed{\mathbf{\frac{2}{15}}}$

Problem set #3

1. Let n be the middle number. We write the product of the three numbers as $(n-1)n(n+1)$. This product is equal to 255 times the middle number, so we write an equation and solve:

$$\begin{aligned}(n-1)n(n+1) &= 255n \\ (n-1)(n+1) &= 255 \\ n^2 - 1 &= 255 \\ n^2 &= 256 \\ n &= \pm 16\end{aligned}$$

Since the three consecutive integers are negative, the value of the middle integer is **-16.**

2. For a triangle, $A = \frac{1}{2}bh$. The area of the changed triangle is $A = \frac{1}{2}(1.2b)(0.9h) = \frac{1}{2}(1.08)bh$, which is an **increase of 8%** from the area of the original triangle.

3. We start by applying the formula of distance = rate times time for the trip into and the trip out of the woods. Going in: $d_1 = 4t$. Coming out: $d_2 = 3(3.5 - t)$. Since the distances are the same, set them equal to each other: $4t = 10.5 - 3t \rightarrow t = 1.5$. Chris hiked for 1.5 hours into the woods at a rate of 4 mph, making the distance 4(1.5) = **6 miles.**

Problem set #4

1. We are free to choose whatever dimensions for the rectangle that we want, so long as the area ends up being 24 square units. The base of the triangle is equal to the base of the rectangle, while the height of the triangle is half the height of the rectangle. Since for the rectangle bh = 24, we use this in the area of the triangle formula: $A = \frac{1}{2}b\left(\frac{1}{2}h\right) = \frac{1}{4}bh = \frac{1}{4}(24) = 6$.

   The area of the triangle is therefore **6 square units.**

2. If 6 chefs can make 30 desserts in 20 minutes, then in the same amount of time, one chef can make 5 desserts, which means that one chef can make one dessert in 4 minutes. If 80 desserts are needed, one chef can make them in 320 minutes. Divide this number by 15 to see how many chefs are needed to make them in 15 minutes: $320 \div 15 = 21.\overline{3}$, so we need a minimum of **22 chefs** to fill this order.

3. Any one side of a triangle must be shorter than the sum of the lengths of the other two sides of the triangle. If we make the $3x - 2$ side be the longest, then we have $2(x + 1) > 3x - 2$, or $x < 4$. In order for the base of the triangle to have a positive length, we need $3x - 2 > 0$, or $x > \frac{2}{3}$. Writing these two inequalities together, we have the range of $\boldsymbol{\frac{2}{3} < x < 4.}$

Team Problem set

1. In the coming years, there will be 365, 366, 365, 365, and 365 days, for a total of 1,826 days to be January 1, 2020. From here, we need to go another 189 days into the future. In the coming months, there are 31, 29, 31, 30, 31, and 30 days for a total of 182 days to be July 1. Seven days more takes us to **July 8, 2020.**

2. Sum the areas of the five equilateral triangles with sides of measure $2\sqrt{3}$ and subtract the sum of the four double counted equilateral triangles with sides of measure $\sqrt{3}$, the heights of each being 3 and $\frac{3}{2}$, respectively: $5\left(\frac{1}{2}\cdot 2\sqrt{3}\cdot 3\right)-4\left(\frac{1}{2}\cdot\sqrt{3}\cdot\frac{3}{2}\right)=15\sqrt{3}-3\sqrt{3}=\boxed{\mathbf{12\sqrt{3}}}$

3. Since 12 is both the median and the unique mode, at least two middle values must be 12. If we had ten 12s, then the last two integers must add up to 24. To keep the range, we must use 6 and 18. With eight 12s, the other four integers must add up to 48. Three 9s and a 21 give us the appropriate sum and range. With six 12s, the remaining six integers must add up to 72, which can be accomplished with five 10s and a 22. 23 cannot be a part of the collection, since this would force the minimum to be at least 11, forcing the collection to be eleven 11s and a 23. So **22** is the largest possible integer value in the collection.

4. The sum of the three angles of a triangle is 180°. $\angle ACB$ and $\angle ACD$ are supplementary, which makes $m\angle ACB = 180-(6x-7)=187-6x$. Add the three angles together to make 180° and solve for x: $2x+(103-x)+(187-6x)=180$

$$110 = 5x$$

$$22 = x$$

$m\angle ACB = 187-6\cdot 22=\boxed{\mathbf{55^\circ}}$

5. There are two types of handshakes to count – man to woman and woman to woman.

   man to woman: There are 15 men shaking hands with 20 woman for a total of 15 × 20 = 300 handshakes.
   woman to woman: Each of the 20 woman shakes hands with 19 women, but each handshake is double counted, so that gives us $\frac{20\times 19}{2}=190$ handshakes.

   Thus there would be a total of $300+190=$ **490 handshakes**.

6. Region A has the numbers 12, 24, and 72, which add up to 108. There are no numbers in Region B. Region C has numbers 2, 3, 6, and 18, which add up to 29. $108+0+29=\boxed{\mathbf{137}}$