

WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET
WISCONSIN MATHEMATICS COUNCIL

February 29-March 4, 2016

Problem Set #1

Score:

(For Scorer's Use Only)

Name: _____

Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

For this first problem set, calculators are not allowed. They may be used for the remainder of the meet only, starting with Problem Set #2.

Answers

1. (1 point)

In simplest form, what is the numerical value of the following expression: $(\sqrt{2016})(\sqrt[3]{2016})(\sqrt[6]{2016})$ _____

2. (3 points)

Express the value of x as a rational number in lowest terms.

$x = \sin^2(10^\circ) + \sin^2(20^\circ) + \sin^2(30^\circ)$
 $+ \sin^2(40^\circ) + \sin^2(50^\circ) + \sin^2(60^\circ)$
 $+ \sin^2(70^\circ) + \sin^2(80^\circ) + \sin^2(90^\circ)$ _____

3. (5 points)

Calculate $(\log_2 x)^4$ if $\log_2(\log_{32} x) = \log_{32}(\log_2 x)$. _____

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Problem Set #2

Score:

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Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

A domino is a rectangular tile divided into 2 squares, each of which contains anywhere from 0 to 6 dots. How many distinct dominoes are there?

2. (3 points)

If i is the imaginary unit, and $\frac{x + yi}{1 + i} = \frac{7}{7 + i}$ where x and y are real numbers, find the value of $x + y$.

3. (5 points)

On a certain unfair 6-sided die, the probability of rolling a particular number is proportional to the number rolled. When rolling two of these dice, what is the probability of rolling a total of 10 or more?

WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET
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Problem Set #3

Score: _____
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Name: _____

Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

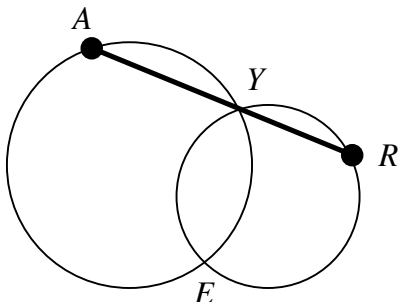
If $\sin x + \cos x = \sqrt{2}$, what is the value of $\sin x \cos x$? _____

2. (3 points)

In the five-digit base 10 numeral $ABCDE$ ($A \neq 0$), different letters do not necessarily represent different digits. If this numeral is the fourth power of an integer, and if $A + C + E = B + D$, find the digit C . _____

3. (5 points)

Two circles intersect at points Y and E . Let \overline{AR} be a segment passing through Y having one endpoint on each circle, as shown. If the circles have radii of 20 and 16, while $YE = 28$, determine the longest possible length of \overline{AR} . _____



WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET
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Problem Set #4

Score:
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Name: _____

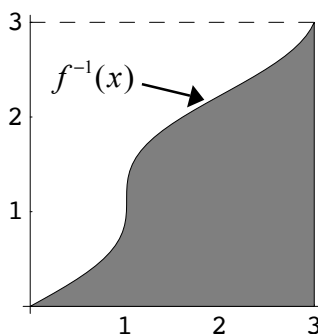
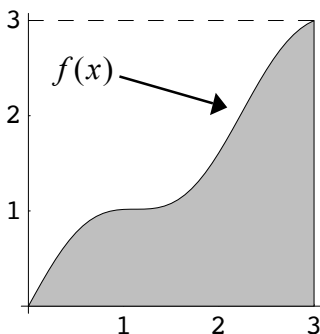
Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

If the light shaded area is 4.2 cm^2 , find the dark shaded area, where $f^{-1}(x)$ is the inverse function of $f(x)$. _____ cm^2



2. (3 points)

Take the set of positive integers and break them up into sets in the following pattern: $\{1\}$, $\{2, 3\}$, $\{4, 5, 6\}$, $\{7, 8, 9, 10\}$,
 What is the sum of the elements in the 35^{th} set? _____

3. (5 points)

Suppose $\{a_n\}$ is a geometric sequence. Let us define

$$S_k = \sum_{n=1}^k a_n \text{ to be the sum of the first } k \text{ terms of this}$$

sequence. If $S_8 = 14$ and $S_{24} = 40$, find S_4 . **A decimal approximation is acceptable for your answer.** _____

WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET
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Team Problem Set (Page 1)

Score:

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Team: _____

Captain: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (10 points)

There are four sets that contain single digits as elements. Any two sets have exactly one element in common. Every element is contained in exactly two sets. No set contains the same element twice. How many elements are in each set?

2. (10 points)

A piece of string is cut into two pieces at a point selected at random. What is the probability that the longer piece of string is at least a times as long as the shorter piece? Write your answer in terms of a .

3. (10 points)

How many different ways are there to score 10 points in basketball, using different orders of 1-, 2-, and 3-point baskets? Note: for example, a 1-point basket followed by a 3-point basket is considered different from a 3-point basket followed by a 1-point basket.

Team Problem Set (Page 2)

4. (10 points)

Consider the 3×3 matrix equation $A^3 = I$, where I is the 3×3 identity matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \text{ Find a solution } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \text{ to the equation } A^3 = I \text{ that}$$

satisfies the following conditions:

- (i) $a = e = i = 0$
- (ii) The product of any four distinct entries in A is zero.
(e.g., $bcdf = 0$)
- (iii) None of the entries equals 1.
- (iv) All non-zero entries are distinct.

5. (10 points)

Reduce the following to a single reduced fraction:

$$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{16}\right)\left(1 - \frac{1}{25}\right) \cdots \left(1 - \frac{1}{2025}\right)$$

6. (10 points)

The lateral surface of a right cone with a right angled vertex is sliced down one side and opened up into a flat sector as seen in the picture. What size angle is formed by the major arc? **A decimal approximation is acceptable. Answer in terms of degrees.**

