

# What is FLUENCY?



*WORK AT YOUR TABLE TO  
CRAFT A WORKING  
DEFINITION OF  
MATHEMATICAL FLUENCY.  
WRITE YOUR DEFINITION ON  
THE BRIGHT PINK SHEET TO  
SHARE OUT WITH THE GROUP.*

# Going Beyond Fast Facts



## A BALANCED APPROACH TO ASSESSING FACT FLUENCY IN GR. 3

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# Compare to Reading...



Think about how you assess a student's reading fluency:

- *Do you **time** them to see how many words they can read correctly in a specified amount of time?*
- *Do you **listen** and **observe** as they read?*
- *Do you **ask** questions to see if they understand what they're reading?*

# Compare to Reading...



Imagine only using timed tests to assess reading fluency...

- Do you **time** them to see how many words they can read correctly in a specified amount of time?
- ~~Do you **listen** and **observe** as they read?~~
- ~~Do you **ask** questions to see if they understand what they're reading?~~

Would you be **confident** in your assessment?

# Memorizing vs Understanding



- Given 10 seconds, can you memorize a list of 12 words?

<b>hair</b>	<b>sat</b>	<b>chair</b>	<b>with</b>
<b>red</b>	<b>girl</b>	<b>curly</b>	<b>a</b>
<b>in</b>	<b>little</b>	<b>the</b>	<b>brown</b>

- Write down as many words as you can remember.

# Memorizing vs Understanding

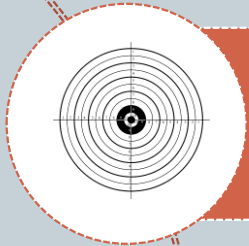


- Given 10 seconds, can you memorize a list of 12 words?

**A girl with curly red hair sat  
in the little brown chair.**

- Write down as many words as you can remember.

# Components of Mathematical Fluency



Accuracy



Efficiency



Flexibility

# Components of Mathematical Fluency



## Accuracy

- correct-ness
- concern about double-checking results

## Efficiency

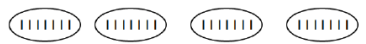

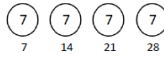
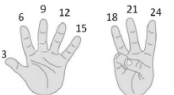
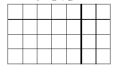
- easily & quickly carry out a strategy
- won't get "lost" in too many steps or lose track of the logic of the strategy



# Components of Mathematical Fluency

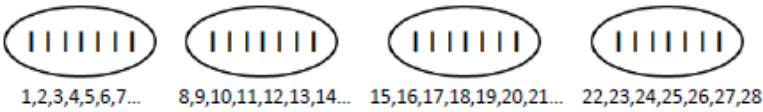
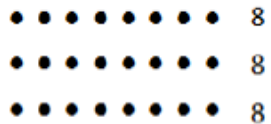
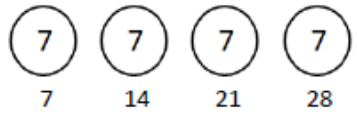
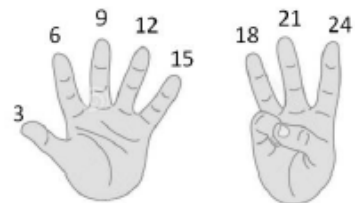
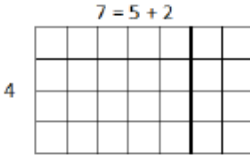
## Flexible Thinking

- comfortable with more than one approach
- use knowledge of number relationships and operations to solve unknown equations
- use more sophisticated methods (appropriate to age & grade)

Levels of Thinking in Multiplication and Division		
	$4 \times 7 = 28$	$24 \div 3 = 8$
<b>Level 1:</b> Model/Count all all (objects or drawings)	<p style="text-align: center;"><i>Count all</i></p>  <p style="text-align: center;">1,2,3,4,5,6,7... 8,9,10,11,12,13,14... 15,16,17,18,19,20,21... 22,23,24,25,26,27,28</p> <p>"I drew 4 groups, with 7 tally marks in each group. When I counted them all, the total was 28, so <math>4 \times 7 = 28</math>."</p>	<p style="text-align: center;"><i>Model all</i></p>  <p>"I made an array with 3 rows of dots and added one dot to each row until I had 24 dots. I counted 8 dots in each row, so <math>24 \div 3 = 8</math>."</p>
<b>Level 2:</b> Repeated counting	<p style="text-align: center;"><i>Skip-count</i></p>  <p>"I skip-counted by 7 four times, and the total was 28. So <math>4 \times 7 = 28</math>."</p>	<p style="text-align: center;"><i>Skip-count</i></p>  <p>"I skip-counted by threes up to 24, keeping track with my fingers, so <math>24 \div 3 = 8</math>."</p>
<b>Level 3:</b> Use properties, patterns	<p style="text-align: center;"><i>Distributive property/Decomposition (mentally or written)</i></p> <p><math>4 \times 7 = 4 \times (5 + 2)</math>  <math>4 \times 7 = (4 \times 5) + (4 \times 2)</math>  <math>4 \times 7 = 20 + 8</math>  <math>4 \times 7 = 28</math></p>  <p style="text-align: center;"><i>Use a fact ahead/behind</i></p> <p>"I know that 5 groups of 7 is 35, and <math>4 \times 7</math> is four groups of 7. If I take one group of 7 away from 35, I get 28. So <math>4 \times 7 = 28</math>."</p>	<p style="text-align: center;"><i>Inverse relationship of multiplication &amp; division</i></p> <p>"I know that <math>3 \times 8 = 24</math>, so <math>24 \div 3</math> must equal 8."</p> <p style="text-align: center;"><i>Decomposition/Solve with a known fact</i></p> <p><math>24 = 12 + 12</math>  <math>12 \div 3 = 4</math></p> <p>"I know that there are two 12s in 24, and I know that <math>12 \div 3 = 4</math>. So I just double the 4 because I need to divide both 12s by three. So <math>24 \div 3</math> equals 8."</p>

Reference: Common Core Standards Writing Team (2011). *Progressions for the Common Core State Standards in Mathematics: K, Counting and Cardinality, K-5 Operations and Algebraic Thinking*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

## Levels of Thinking in Multiplication and Division

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<p><b>Level 1:</b></p> <p>Model/Count all (objects or drawings)</p>	<p><i>Count all</i></p>  <p>1,2,3,4,5,6,7...    8,9,10,11,12,13,14...    15,16,17,18,19,20,21...    22,23,24,25,26,27,28</p> <p>"I drew 4 groups, with 7 tally marks in each group. When I counted them all, the total was 28, so <math>4 \times 7 = 28</math>."</p>	<p><i>Model all</i></p>  <p>"I made an array with 3 rows of dots and added one dot to each row until I had 24 dots. I counted 8 dots in each row, so <math>24 \div 3 = 8</math>."</p>
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# Why focus on fluency?



*"Math fact fluency leads to higher order mathematics.*

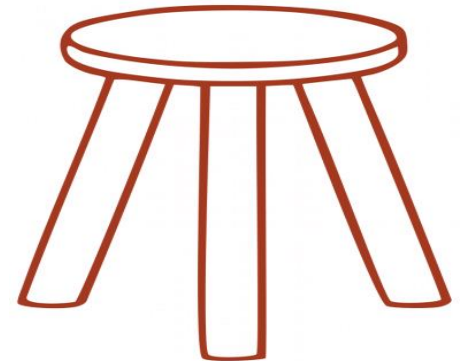
*Through automaticity students free up their working memory and can devote it to problem solving and learning new concepts and skills (Geary, 1994)."*

# Common Core State Standards—Mathematics

## Instructional Shift:

# Rigor

- Conceptual understanding
- Procedural skill and **fluency**
- Application of skills in problem solving situations



# Common Core State Standards—Mathematics

## Required Fluencies:

Grade	Standard	Required Fluency
K	K.OA.5	Add/subtract within 5
1	1.OA.6	Add/subtract within 10
2	2.OA.2 2.NBT.5	Add/subtract within 20 (know single-digit sums from memory) Add/subtract within 100
3	3.OA.7	Multiply/divide within 100 (know single-digit products from memory) Add/subtract within 1000
4	4.NBT.4	Add/subtract within 1,000,000
5	5.NBT.5	Multi-digit multiplication
6	6.NS.2,3	Multi-digit division Multi-digit decimal operations

How can we get useful information about students' mathematical fluency?



April 2014

# Assessing Basic Fact *Fluency*

Have you had it with timed tests, which present a number of concerns and limitations? Try a variety of alternative assessments from this sampling that allows teachers to accurately and appropriately measure children's fact fluency.

By Gina Kling and Jennifer M. Bay-Williams



***Assessing Fluency:  
A Case Study***



## Timed Tests

- Using the timed test, what can you tell about students' fluency?
- What are next instructional steps for these students based on this assessment?





# Journaling

- Pose meaningful writing prompts
- Assess flexibility & understanding of strategy selection
- Engage students in self-reflection and self-monitoring

- Using the journal responses, what can you tell about students' fluency?
- What are next instructional steps for students based on this assessment?

# Interviews

- Opportunity to listen to children explain what they know (formal or informal)
- Teacher is able to ask follow-up and clarifying questions
- As a formative assessment, can inform instruction for individual or class

Student Name: Sky

Date: \_\_\_\_\_

## 3rd Grade Fluency Interview: X3 / ÷3

		Strategy		
		Level 3: Decompose or Convert to an Easier Equivalent Problem	Level 2: Skip Count	Level 1: Direct Model (Draw a picture, array, fingers, hundred chart, etc.)
8	$6 \times 3 =$	• Recall • Strategy: multiplied 2 counters	• Mentally • Fingers	• Tool:
2	$4 \times 3 =$	• Recall • Strategy: multiplied 3, 4 times	• Mentally • Fingers	• Tool:
9		• Recall • Strategy:	• Mentally • Fingers	• Tool: Counted by 2's until started
2		• Recall • Strategy:	• Mentally • Fingers	• Tool: Counted by 2's until reached 6
3	$\square \times 7 = 21$	• Recall • Strategy:	• Mentally • Fingers	• Tool:
2		• Recall • Strategy:	• Mentally • Fingers	• Tool: Counted by 3's until reached 6

## 3rd Grade Fluency Interview: X4 / ÷4

		Strategy		
		Level 3: Decompose or Convert to an Easier Equivalent Problem	Level 2: Skip Count	Level 1: Direct Model (Draw a picture, array, fingers, hundred chart, etc.)
2	$8 \times 4 =$	• Recall • Strategy:	• Mentally • Fingers	• Tool: Draw groups of 4's until reach 8 groups
4	$16 \div 4 =$	• Recall • Strategy:	• Mentally • Fingers	• Tool: Draw groups of 4 until reached 16
4	$6 \times 4 =$	• Recall • Strategy:	• Mentally • Fingers	• Tool: Draw 4 groups of 6
3		• Recall • Strategy:	• Mentally • Fingers	• Tool:
7	$7 \times 4 = 28$	• Recall • Strategy:	• Mentally • Fingers	• Tool:
6	$12 \div 4 = 3$	• Recall • Strategy:	• Mentally • Fingers	• Tool: Draw 4 groups of 3

Demonstrates Multiplication/Division fluency using this strategy:

Foundational Facts	Derived Facts	X3	SCFc	SCFv	X4	FC	FC	Fluency Codes							
								R:c	R:v	EE:c	EE:v	SC:c	SC:v	SC-F:c	SC-F:v

□ Continue to work on fluency flexibility using missing factors and dividends



- Using observational data,

Student	Pictures or Tally marks	Fingers Skip Count	Array	Distributive Property	Known Fact
Sam	6+	4,3,2			0,1,5,10
Elijah		8	7+	6,9	0,1,2,3, 4,5,10
Summer	2-10				1



## Observation

- Focused observations of students at work provide insights into students' strategies
- Can be recorded on a clipboard as students work

- Using observational data, what can you tell about students' fluency?

Student	Pictures or Tally marks	Fingers Skip Count	Array	Distributive Property	Known Fact
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Summer	2-10				1

- What are next instructional steps for students based on this assessment?

# Debrief



**WHAT ARE THE ADVANTAGES  
AND DISADVANTAGES OF EACH  
OF THE ASSESSMENT METHODS  
THAT WE EXAMINED TODAY?**

# Instructional Implications

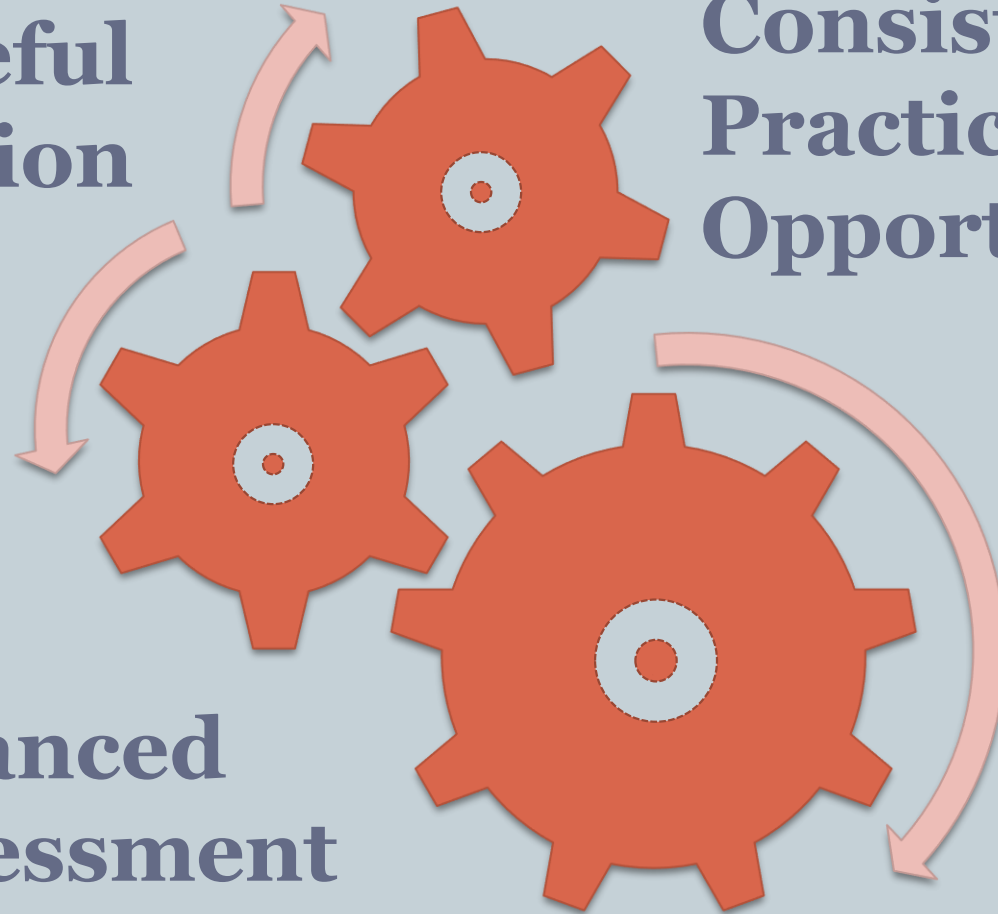


# Fluency is an endpoint



**Purposeful  
Instruction**

**Consistent  
Practice  
Opportunities**



**Balanced  
Assessment**

# The Impact of Instruction

## SLO Fluency Progress: Class A

	X1	X2	X5	X10	X0	X3	X4	X9	X6	X7	X8
A'ohnasti	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc
Breylon	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc
Teejaylah	Rc	Rc	Rc	Rc	R✓	R✓	R✓				
Payge	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc
Jacari	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc
Antanija	Rc	Rc	Rc	Rc	Rc	Rc	Rc	SCFc	Rc	SCF✓	Rc
Salista	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc
Dallas	Rc	Rc	Rc	Rc	Rc	Rc	Rc	SCFc	Rc	Rc	Rc
Jaelynn	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	SCc
Omarion	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc
Darius	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc
Freeyon	Tc	Tc					SCFc				
Champion	Rc	Rc	Rc	R✓			Rc	R✓	Rc	SC✓	
Angel	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc
India	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc
Demetrius	Rc	SC✓	Rc	R✓							
Kevon	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc			

## SLO Fluency Progress: Class B

	X1	X2	X5	X10	X0	X3	X4	X9	X6	X7	X8
Joey	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc	Rc
Lashunda	Rc	Rc	Rc	Rc	Rc	F✓	F✓	R✓			
Finesse	Rc	SCFc	SCF✓	Fc	Rc						
Darnell	Rc	Rc	Rc	Rc	Rc						
Ladarus	Rc	Rc	F✓	SC✓	R✓						
Jade	Rc	Rc	Rc	F✓	R✓						
Cloteria	R✓	R✓			R✓						
Alani	Rc	Rc	SCFc	Rc	R✓						
Romero	Rc	Rc	Rc	F✓	Rc						
Tayshawn	Rc	Rc	Rc	Rc	Rc	Rc	Rc	R✓	R✓	SCc	SCc
Jalen	Rc	SCF✓			SC✓						
Makayla	Rc	Rc	Rc	Rc	R✓						
Rilya	Rc	Rc	R✓	Rc	Rc	R✓	R✓				
Kamonte	Rc	Rc	SCFc	Rc	R✓						
Valencia	Rc	Rc	Rc	Rc	Rc	Rc					

September   October   December   March   April   May

Met Goal:   September   November   January   March



# Considerations...



- Think about your current assessments. Is there a balance of each of the components—accuracy, efficiency, and flexibility? If not, how might you alter your assessment practices to have a better balance?
- As you reflect on your students' fact fluency, what would you like to know more about? Which of the tools discussed today might help you gain this information?
- Discuss your thoughts on the issue of timed tests. What might you do as a teacher to avoid potential negative impacts with your students?
- How might we help parents better understand the three components of fluency?



## *Thank you for attending our session!*

### *References:*

Common Core Standards Writing Team (2011). *Progressions for the Common Core State Standards in Mathematics, K-2 Operations and Algebraic Thinking*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Geary, D.C. (1994). *Children's mathematical development: Research and practical applications*. Washington, DC: American Psychological Association.

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Kling, G. & Bay-Williams, J.M. (2014). Assessing basic fact fluency. *Teaching Children Mathematics*, 20(8), p. 489-497.

O'Connell, S. & SanGiovanni, J. (2011). *Mastering the basic math facts in addition and subtraction*. Portsmouth, NH: Heinemann.