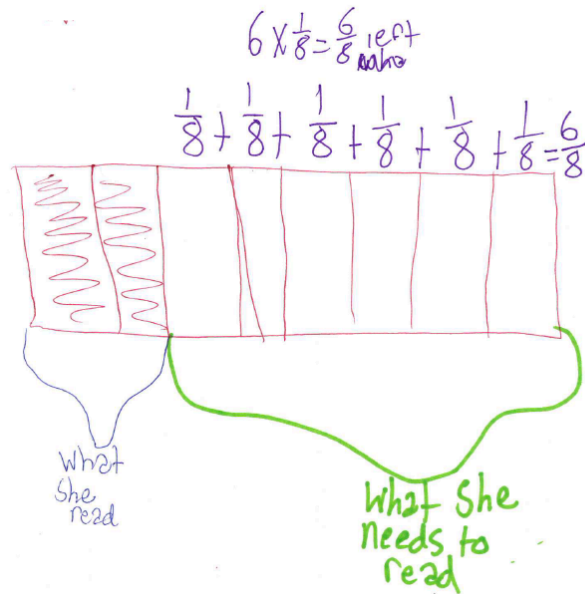


Using Unit Fraction Reasoning to Apply and Extend Understanding of Operations with Fractions



Friday, May 6, 2016
Wisconsin
Mathematics Council
Annual Conference

Introductions

Eric KanTERS

Brookhill Institute of Mathematics

Liz Cutter

Edgerton Elementary, Whitnall School District

UWM Pathways Graduate Student

Session Goals

We are learning to:

- Be able to define a unit fraction.
- Be able to explain how non-unit fractions are built out of unit fractions.
- Understand how unit fraction understanding can help students operate on fractions.

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.**
- 3. Construct viable arguments and critique the reasoning of others.**
- 4. Model with mathematics.**
- 5. Use appropriate tools strategically.**
- 6. Attend to precision.**
- 7. Look for and make use of structure.**
- 8. Look for and express regularity in repeated reasoning.**

Pick up strips and fold:

Fold one YELLOW strip into halves.

Fold one YELLOW strip into fourths.

Fold one YELLOW strip into eighths.

Fold one BLUE strip into thirds.

Fold one BLUE strip into sixths.

Discuss what you are noticing mathematically as you fold!

Your white strip will be your “whole” or “one.”

Focus on Unit Fractions

- Fold each fraction strip to show only one “unit” of each strip.
- Arrange these strips in a way that makes sense to you.
- What are some observations you can make about fractions based on what you have shown here?

How would you describe a unit fraction?

Standard 3.NF.A.1 Unit Fractions

- Read the first part of the standard.

Practice with your shoulder partner using the language of this standard to describe your unit fractions.

3.NF.A. 1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

Unit Fractions as Building Blocks

- Fold your eighths strip to show $\frac{5}{8}$.

→ How do you see this fraction as composed of unit fractions?

Unit Fractions as Building Blocks

- Read the second part of the standard and practice using the language of the standard to describe non-unit fractions.

$$\frac{2}{3}$$

$$\frac{4}{4}$$

$$1\frac{2}{6}$$

$$\frac{5}{2}$$

3.NF.A. 1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; **understand a fraction a/b as the quantity formed by a parts of size $1/b$.**

Solve & explain your reasoning

- Sam walks $\frac{2}{6}$ of a mile to get to school, Hattie walks $\frac{2}{8}$ of a mile to get to school, and Edgar walks $\frac{2}{3}$ of a mile to get to school. Who walks the farthest distance? Who walks the shortest distance?



Standards 3.NF.A.3.d and 4.NF.A.2

3.NF.A.3.d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

How can unit fraction understanding help students to operate on fractions?



What does $\frac{5}{4}$ mean?

$$\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. 4.NF.A.3 Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2 \frac{1}{8} = 1 + \frac{1}{8}$

Use Fraction Strips to Represent and Solve

- Eric's family drank $\frac{3}{4}$ of a gallon of milk on Saturday. Then on Sunday they drank another $\frac{7}{8}$ of a gallon of milk. How many gallons did they drink over the weekend?



Solve and use fraction strips or a picture to represent your thinking

If each person at a party will eat $\frac{3}{8}$ pounds of pasta, and there will be 5 people at the party, how many pounds of pasta will be needed? Between what 2 whole numbers does your answer lie?



Unit Fractions and Multiplication

- In the implied language of 4.NF.B.4.a:

$$\frac{3}{8} = 3 \text{ groups of } \frac{1}{8} = \frac{3}{8} = 3 \times \frac{1}{8}$$

4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of $1/b$. For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

$$5 \times \frac{3}{8} = 5 \times \left(3 \times \frac{1}{8} \right) = (5 \times 3) \times \frac{1}{8} = 15 \times \frac{1}{8}$$

5 groups of $\frac{3}{8}$ is the same as 5 groups of 3 groups of $\frac{1}{8}$ or 15 groups of $\frac{1}{8}$

4.NF.B.4b Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)*

Solve and use fraction strips or a picture to represent your thinking

If each person at a party will eat $\frac{3}{8}$ pounds of pasta, and there will be 5 people at the party, how many pounds of pasta will be needed? Between what 2 whole numbers does your answer lie?



5th Grade Division Standards

5NF.B7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.¹

- a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.*
- b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.*
- c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$ -cup servings are in 2 cups of raisins?*

Use fraction strips or draw a picture to solve the following problems:

$$\frac{1}{6} \div 3 =$$

$$3 \div \frac{1}{6} =$$

How will a unit fraction approach help your students understand operations on fractions?



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Questions???

Thank You for Coming!

Eric Kanters

eric.kanters@brookhillmath.org

Liz Cutter

ecutter@whitnall.com