Multiplication Strings Task

1. Solve this set of multiplication problems. Each time you solve a problem, try to use the previous problem to help you solve it. Note your reasoning for each step.

   8 x 4 = ____

   8 x 8 = ____

   8 x 16 = ____

   8 x 32 = ____

   8 x 64 = ____

2. Review the sequence of equations, what are some patterns you notice in the factors and how this impacts the related solutions?

3. To solve 8 x 8, Nicholas explained that he “knew that 8 times 4 would be 32” and then he added it to another 32. Show how you could represent his reasoning with an area model. How does his strategy utilize properties of the operations?

Connections to the *Common Core State Standards for Mathematics* (CCSSM)
"The Case of Katherine Casey and the Multiplication Strings Task"

Standards for Mathematical Content Domain: Operations and Algebraic Thinking (OA)
Cluster: Understand properties of multiplication and the relationship between multiplication and division.
3.OA.B.5 Apply properties of operations as strategies to multiply and divide.[1] Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56. (Distributive property.)
Cluster: Solve problems involving the four operations, and identify and explain patterns in arithmetic.
3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

1 Students need not use formal terms for these properties.

Standards for Mathematical Content Domain: Measurement and Data (MD)
Cluster: Relate area to the operations of multiplication and addition.
3.MD.C.7c Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a x b* and *a x c*. Use area models to represent the distributive property in mathematical reasoning.
3.MD.C.7d Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Standards for Mathematical Content Domain: Number and Operations in Base Ten (NBT)
Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Source: National Governors Association Center for Best Practices and Council of Chief State School Officers. (2014). *Common core state standards for mathematics*. Washington, DC: Authors. Retrieved from http://www.corestandards.org/Math/

Standards for Mathematical Practice (SMP)
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

SMP 7. Look for and make use of structure.

Mathematically proficient students at the elementary grades use structures such as place value, the properties of operations, other generalizations about the behavior of the operations (for example, the less you subtract, the greater the difference), and attributes of shapes to solve problems. In many cases, they have identified and described these structures through repeated reasoning (MP.8). For example, when younger students recognize that adding 1 results in the next counting number, they are identifying the basic structure of whole numbers. When older students calculate 16 x 9, they might apply the structure of place value and the distributive property to find the product: 16 x 9 = (10 + 6) x 9 = (10 x 9) + (6 x 9). To determine the volume of a 3 x 4 x 5 rectangular prism, students might see the structure of the prism as five layers of 3 x 4 arrays of cubes.

SMP 8. Look for and express regularity in repeated reasoning.

Mathematically proficient students at the elementary grades look for regularities as they solve multiple related problems, then identify and describe these regularities. For example, students might notice a pattern in the change to the product when a factor is increased by 1: 5 x 7 = 35 and 5 x 8 = 40, the product changes by 5; 9 x 4 = 36 and 10 x 4 = 40, the product changes by 4. Students might then express this regularity by saying something like, "When you change one factor by 1, the product increases by the other factor." Younger students might notice that when tossing two-color counters to find combinations of a given number, they always get what they call "opposites"—when tossing 6 counters, they get 2 red, 4 yellow and 4 red, 2 yellow and when tossing 4 counters, they get 1 red, 3 yellow and 3 red, 1 yellow. Mathematically proficient students formulate conjectures about what they notice, for example, that when 1 is added to a factor, the product increases by the other factor; or that, whenever they toss counters, for each combination that comes up, its "opposite" can also come up. As students practice articulating their observations, they learn to communicate with greater precision (MP.6). As they explain why these generalizations must be true, they construct, critique, and compare arguments (MP.3).

Source: Illustrative Mathematics. (2014, February 12). Standards for Mathematical Practice: Commentary and Elaborations for K–5. Tucson, AZ. Retrieved from http://commoncoretools.me/wp-content/uploads/2014/02/Elaborations.pdf (p. 18-19)

Multiplication Strings

Teacher: Katherine Casey

District: New York Community School District 2

Grade: 4

Teacher: All right, let's start with this first one. Thumbs up, 8 times 4. Thumbs up if you know it, 8 times 4. Uh, Ambrese? *(8 x 4 = _ is recorded on the chalkboard.)*

Student: 32.

Teacher: Okay. How'd you know it? *(32 is recorded as the product of 8 x 4.)*

Student: First cause I counted by 4s.

Teacher: And what did that sound like when you counted by 4s?

Student: 4, 8, 6…wait, 6… *(The sequence of numbers is recorded on the chalkboard.)*

Student: 12.

Student: 12, 16, 20, 24, 28, 32.

Teacher: Okay. Anyone else know it a different way? Zina?

Student: I added 8 and 8 is 16, then I added 16 plus 16 equals 32. *(8 + 8 = 16 and 16 + 16 = 32 is recorded on the chalkboard.)*

Teacher: So you knew 8 plus 8 is 16, and then 16 plus 16 equals 32. Okay, great. Um, next one. You thinking about it? Thinking hard? Uh, Nicholas? *(8 x 8 = _ is recorded on the chalkboard.)*

Student: 64. *(The product of 8 x 8 is recorded.)*

Teacher: And how'd you know that? Could we have all eyes on Nicholas when he's speaking, so you can listen in to his thinking?

Student: Cause I knew that 8 times 4 would be 32.

Teacher: Okay, so let me show you that. If I had to draw an array, like we've been working on in multiplication, I would draw it like this, right? 8 times 4, just like all of our posters have and all your array cards, 8 times 4 is 32. *(An 8 by 4 rectangular model is drawn and the dimensions of 8 and 4 labeled. The product 32 is recorded in the center of the rectangle.)*

Student: And then plus 32 and then equals 64.

Teacher: And so then you doubled that, you put on another 32. Four more groups of 8 equals 32, and when you add that together it’s 64. Cause those are 8, times 8, is 64. Neat. Did anyone else have a, um, way of thinking about it? Um, Allison?

Student: Count 8 eight times.

Teacher: Okay, so you used the skip counting. I wanna go back to, um, what Nicholas explained about his strategy. Could anyone put into their own words what Nicholas did when he was solving this problem? Who put that into their own words? Again, all I need is the thumbs. Um, Jasmine, try to put it into your own words.

Student: It seems like he added 32 plus 32. Cause he knew that, um, 8 times 4 was 32. So he just had to add another 4 and he, he just did 32 plus 32.

Teacher: So he already knew that 8 times 4 is 32, and then you said he added another 4? Isn’t 32 plus 4, 36? *(8 x 4 = 32 is recorded on the board.)*

Student: He added, no he added like another set of…I mean he, yeah, he added like another…

Student: …set of 4…

Student: yeah, set of 4…

Teacher: So he added another set of 4s.

Student: No he added, yeah. And that, he probably did 8 times 4 two times, probably.

Teacher: He did do 8 times 4 two times. *(Points to the dimensions of the open area model.)* And that’s what that array shows us. We started with the 8 times 4 is 32.

Student: Then, like, he probably added the 32…

Teacher: Exactly. Four more groups of 8 is 32 and that equals 64 altogether. *(Points to the dimensions of the second rectangle.)* Hmmm, if this is this, what would this be? 8 times 16. Thumbs, thumbs. *(8 x 16 = _ is recorded on the chalkboard.)*

Student: Oh, I know.

Teacher: Oooh, Muhammed.

Student: 128.

Teacher: 128! How'd you get that?

Student: Um…

Teacher: Let me erase something. How'd you get that, Muhammed?

Student: I add 64 plus 64.

Teacher: So you got 128 by adding 64 and 64 together to equal 128. *(128 is recorded as the product of 16 x 8. Teacher points to the product of 8 x 8 = 64.)* My question immediately is going to be why? How'd you know to do that?

Student: Because I…I…I noticed that 8 plus 8 equals 16…

Teacher: Okay, so you took the 16 and…

Student: And then…

Teacher: Hang on a second, and you thought of that as being 8 plus 8. Great. *(8 + 8 is recorded on the chalkboard.)*

Student: Yeah, and then…and then, um, when I saw it's 16, I just plus it again. I plussed the 64 again. Plus I noticed…I noticed that is plus, um, 8 more, and then I just plussed 64 plus 64 equals 128.

Teacher: How many people followed what Muhammed was just saying? Did you hear what he said? Polina, did you listen in?

Student: Yeah.

Teacher: Okay, can you say it really loudly and slowly?

Student: Muhammed said that, uh, he knew that, um, 8 times 16 equals 128, uh, because, before that, he saw that 8 times…8 plus 8 equals 16. And, uh, and that's the second number. And uh, if 8 times 8 equals 64…

Teacher: Okay, slow down. So if 8...we know that 8 times 8 equals 64, cause we saw that up here, 8 times 8 is 64. Okay. *(An 8 x 8 square is drawn and the dimensions are labeled. The product of 8 x 8 is noted in the center of the square. The teacher points to the previous area model showing 4 x 8 and 4 x 8.)*

Student: And now, um, he adds another 64.

Teacher: Now he adds on another 64 because before he said, “Well, I know 16 has 2 groups of 8, so 8 times 8 is 64 here. One more group of 8 times 8 is 64,” and Muhammed, how much is this 8 plus 8 up here? *(Another 8 x 8 square is drawn and connected to the previous 8 x 8 square. The teacher points the multiplication equations written previously (8 x 8) and then records the dimensions of the square and writes the product 64 in the center of the square.)*

Student: 16.

Teacher: There’s your 16. So if 8 times 8 is 64, and another 8 times 8 is 64, 16 times 8 is…add those together, 128. Okay? Yes, Joseph? *(The teacher points to the 8 and 8, connects the 8s and writes 16 over the amounts.)*

Student: Um, I was looking at 8 times 4, 8 times 8, and 8 times 16. There’s a pattern.

Teacher: Oh!

Student: Because every time we started out with 32, and then went with 64, and there was another 32, and then we went with 64, and there was another 64. So…

Teacher: Say more.

Student: Every time you add, it’s the number before, you answered.

Teacher: Yeah. Adrian, did you want to say something about that?

Student: No, I wanted to say another pattern.

Teacher: Let’s focus on what Joseph was just talking about with his pattern. He’s noticing, guys look up here, look at these numbers, everyone. Joseph is noticing something about 32 and 32 more is 64 and then 64 and 64 more is 128. *(The teacher points to the product of each of the equations.)* He’s noticing something happening with these numbers. Let’s really, let’s spend some time thinking about that.

Eyes back this way. I listened in on a couple of conversations. Um, Jeffrey and Aaron, I’d like you, cause I listened in to your conversation, I’d like you to share what you notice is happening with these numbers.

Student: We noticed that, the um, the questions are doubling…

Student: …and the answers.

Teacher: To end this string, I heard someone say, “I know what would come next.” What do you think would come next in this number string from the 8 by 16, if I use this doubling? Think about that for a second.

So the big idea, guys, put your thumbs down, let’s listen in to this big idea, cause we’re gonna follow this as we’re working with our arrays. We have this big idea going here, that if one factor doubles, this one’s staying the same the whole time, but if this one factor doubles, the answer’s gonna double. *(The teacher points to the factors in one equation as she talks about doubling the factor she then points to the product of the next equation in the string.)*

[End of audio]

Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. *Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.*

Implement tasks that promote reasoning and problem solving. *Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.*

Use and connect mathematical representations. *Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.*

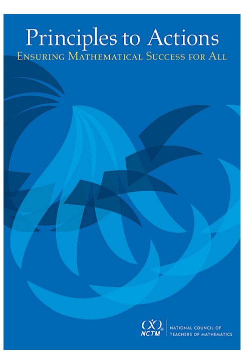
Facilitate meaningful mathematical discourse. *Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.*

Pose purposeful questions. *Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.*

Build procedural fluency from conceptual understanding. *Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.*

Support productive struggle in learning mathematics. *Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.*

Elicit and use evidence of student thinking. *Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.*



National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

Writing Team: Steve Leinwand, Daniel J. Brahier, DeAnn Huinker, Robert Q. Berry III, Frederick L. Dillon, Matthew R. Larson, Miriam A. Leiva, W. Gary Martin, and Margaret S. Smith.

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