WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET

WISCONSIN MATHEMATICS COUNCIL

March 4 – 8, 2019

Problem Set #1 Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

For this first problem set, calculators are <u>not</u> allowed. They may be used for the remainder of the meet only, starting with Problem Set #2.

Answers

1. (1 point)

   Let $a = 1-\sqrt{3}$ and $b = 1+\sqrt{3}$. The expression $\frac{a}{b}+\frac{b}{a}$ equals what integer? _____

2. (3 points)

   A bag is holding 2019 tickets numbered 1 through 2019. What is the probability that the first 5 tickets drawn from the bag, without replacement, are in increasing order? _____

3. (5 points)

   Find the shortest distance between these two curves: _____

   $C_1: x^2+y^2-2x-2y-23=0$

   $C_2: 9x^2+4y^2-18x-27=0$

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Problem Set #2

Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

   400 delegates are attending a convention. 300 of them are Libertarian and the rest are Socialists. 150 of all of the delegates are vegetarians. Of those who are not vegetarians, 200 of them are Libertarians. How many delegates are both Socialist and vegetarian?

   _____

2. (3 points)

   Solve this equation: $\log(x+1)+\log(x-1)=1$

   _____

3. (5 points)

   Ron tosses a fair coin 10 times, ending up with 6 heads and 4 tails. What is the probability that, after the first 5 tosses, he had 3 heads and 2 tails?

   _____

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Problem Set #3 Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

   If you are standing 20 feet away from a tree, and the angle of elevation from your feet to the top of the tree is 35°, what is the height of the tree, rounded to the nearest inch?

   _____ ft _____ in

2. (3 points)

   Pirate Jack Sparrow decides to start a retirement fund for himself. His Edward Jones advisor suggests that after each successful raid of English ports in the Spanish Main, he should bury some of his treasure. Knowing that pirates do not have long life spans while pirating, he decides to stow away 50% more treasure than the previous bury, to quickly build up his net worth. After six buries, he wants to dig up all of his treasure and retire to Alaska. If he digs up 19,950 gold doubloons in total, how many coins did he bury the very first time?

   _____

3. (5 points)

   Let $f(n)=\frac{n+2019}{3n+1}$ for all positive integers n. If $f(N)$ is a positive integer for some positive integer N, find the largest integer that N can be.

   _____

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Problem Set #4 Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

   What is the difference between the sum of all eight positive integral divisors of 66 and the sum of all eight positive integral divisors of 70? _____

2. (3 points)

   A sphere is placed inside a cone so that the sphere is tangent to the side of the cone and also tangent to the base of the cone. If the radius of the cone is 5, and the height of the cone is 12, what is the radius of the sphere? _____

3. (5 points)

   Functions $f(x)$ and $g(x)$ are defined for all real numbers so that they satisfy the following condition: For any real numbers x and y, $f(x+g(y))=4x+y+7$. Find an explicit expression for $g(x+f(y))$. _____

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Team Problem Set (Page 1) Score: (For Scorer's Use Only)

Team: _____ Captain: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

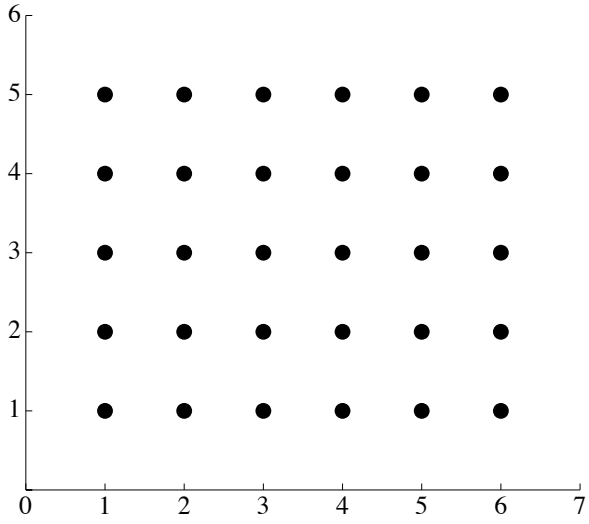
Answers

1. (10 points)

   Let M be the set of real numbers that can be written as repeating decimals of the form $0.\overline{abc}$, where a, b, and c are distinct digits. What is the sum of all of the elements of M? _____

2. (10 points)

   Draw a polygon on the coordinate grid to the right whose perimeter is 12 units and area is 4 square units. The polygon's vertices must be lattice points, i.e. have integer coordinates. Use the dots on the grid for the polygon's vertices.



3. Solve the following equation for $x > 0$ in exact form: _____

$$e^{xe^{xe^{x\cdots}}} = 2019$$

Team Problem Set (Page 2)

4. (10 points)

   A sequence of 2019 digits has the property that each pair of consecutive digits forms a two-digit number that is divisible by either 17 or 23. If the first digit is 6, and the final digit is not 3, then what is the final digit?

   _____

5. (10 points)

   A committee makes up a list of problems for a math contest involving 9^{th}, 10^{th}, 11^{th}, and 12^{th} grades. Committee members decide that the list for each grade will comprise twelve problems, such that exactly seven of them do not appear on the list for any other grade. What is the greatest number of distinct problems that could be included in the contest?

   _____

6. (10 points)

   Here is a rectangle that has been divided into eleven squares of various sizes. The smallest square has an area of $81 cm^2$. Find the area of the largest square.

   _____