Productive Struggle & Problem Solving

Grades 3-5

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What is Productive Struggle?

- Productive: adjective

  having the power of producing, generative, **creative**

- Struggle: verb

  1. to contend with an adversary or **opposing force**

  2. to contend resolutely (determined or set in purpose)

  with a task, problem

  3. to advance with **violent effort**

What is Productive Struggle?

"Effective mathematics teaching supports students in struggling productively as they learn mathematics. Such instruction embraces a view of students' struggles as opportunities for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions. (NCTM, Principles to Actions, p. 48)

- The process is just as, or more important, than the outcome.

Expectations for Students	Teacher actions to support students	Classroom based indicators of success
Most tasks that promote reasoning and problem solving take time to solve, and frustration may occur, but perseverance in the face of initial difficulty is important.	Use tasks that promote reasoning and problem solving; explicitly encourage students to persevere; find ways to support students without removing all the challenges in a task.	Students are engaged in the tasks and do not give up. The teacher supports students when they are "stuck" but does so in a way that keeps the thinking and reasoning at a high level.
Correct solutions are important, but so is being able to explain and discuss how one thought about and solved particular tasks.	Asks students to explain and justify how they solved a task. Value the quality of the explanation as much as the final solution.	Students explain how they solved a task and provide mathematical justifications for their reasoning.
Everyone has a responsibility and an obligation to make sense of mathematics by asking questions of peers and the teacher when he or she does not understand.	Give students the opportunity to discuss and determine the validity and appropriateness of strategies and solutions.	Students question and critique the reasoning of their peers and reflect on their own understanding.

Expectations for Students	Teacher actions to support students	Classroom based indicators of success
Diagrams, sketches, and hands on material are important tools to use in making sense of tasks.	Give students access to tools that will support their thinking process.	Students are able to use tools to solve tasks that they cannot solve without them.
Communicating about ones thinking during a task makes it possible for others to help that person make progress on the task.	Ask students to explain their thinking and pose questions that are based on students' reasoning, rather than on the ay that the teacher is thinking about the task.	Students explain their thinking about a task to their peers and the teacher. The teacher asks probing questions based on the students' thinking.

NCTM: Principles to Actions, p. 49, Fig. 20

Group Task

- Create a title for your topic. Did any themes emerge?
- What are the essential elements?
- What is different from what we typically see in our classrooms?
- Prepare to share the "big ideas"

NCTM Mathematics Teaching Practices– Which are present?

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Things to consider

- What traditionally occurs in our classrooms when a student is struggling?
- What happens when we rescue a student?
- Are these behaviors difficult for teachers to change?
- How do students respond when we don't rescue?
- Are we asking students to think?

Levels of Cognitive Demand

- Handout: Figures 3 & 4
- Lower Level Demand
  - Memorization
  - Procedures without Connections
- Higher Level Demand
  - Procedures with Connections
  - Doing Mathematics

Maintaining Cognitive Demand: Be Intentional and Deliberate

- Planning
- Implementation
- Assessment

Looks like…Sounds Like

Support productive struggle in learning mathematics Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle. Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. Helping one another without telling their classmates what the answer is or how to solve the problem.

NCTM Principles to Actions, p 52

Example: Task

Lower Cognitive Demand	Higher Cognitive Demand
On a double decker bus, 9 people are in the first level and 10 are on the second level. How many people are on the bus?	There are 19 people on a double decker bus. How many people might be on each level of the bus?

NCTM, Teaching Children Mathematics

Questions

Lower Cognitive Demand	Higher Cognitive Demand
What is your answer? Where is your work? How did you get 19?	Have you found all of the possible combinations? A student once told me there were only 9 possible combinations? Were they correct? How do you know?

"A high cognitive demand question is one that invites students to explain their thinking, make new connections, describe their process, and critique others. Questions that maintain high cognitive demand engage students in making more sense of the mathematics."

NCTM, Teaching Children Mathematics

Make Sense and Persevere

Lower Cognitive Demand	Higher Cognitive Demand
Students have to take 2 numbers and add them.	Students have to add multiple combinations. Students have to determine whether they have located all possible combinations. Students have to figure out what it means to have 19 students on a double decker bus.

NCTM, Teaching Children Mathematics

Make Viable Arguments and Critique the Reasoning of Others

Lower Cognitive Demand	Higher Cognitive Demand
Some opportunity to explain how the student knows the answer.	Opportunities to hear about strategies for calculating and strategies for deciding whether all possibilities have been found.

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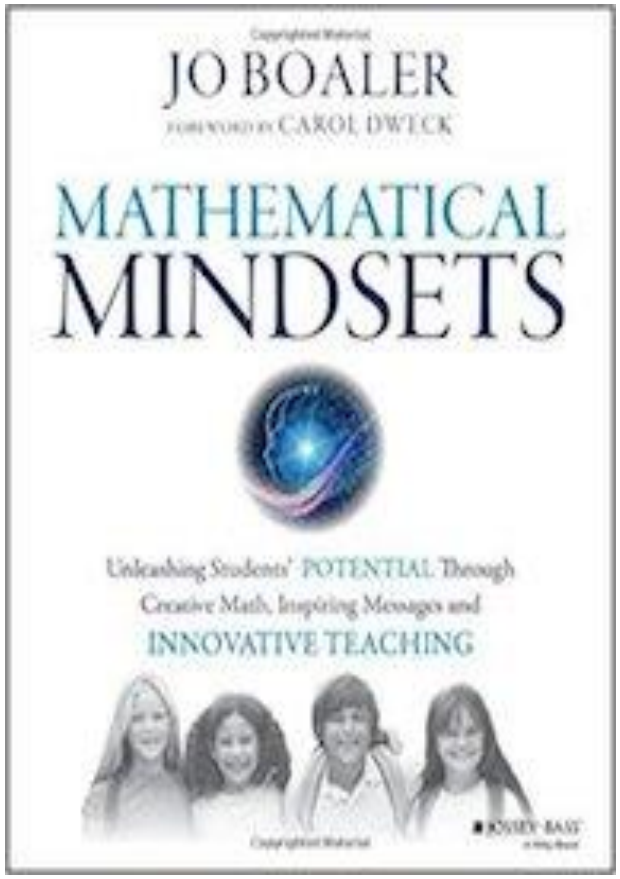
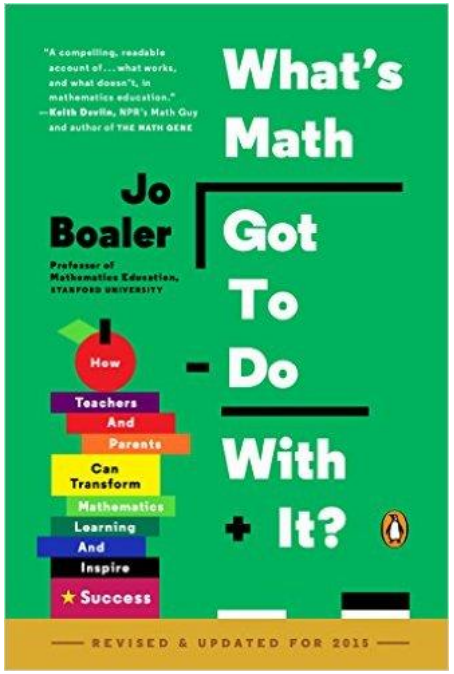
Growth vs. Fixed Mindset





Growth vs. Fixed Mindset

- https://www.youtube.com/watch?v=Xs9aGVUZ3YA
- Handout: Beliefs about access and equity in mathematics

Classroom Activities for Growth Mindset

- 4 Boxes, 4 minutes
  - 1 minute– list all of the things that have been really hard for you in math, this year, last year, back in kindergarten (these can be things you have mastered now, but think back to the really hard times)
  - 1 minute – list all of the statements you say in your head when things get hard in MATH
  - 1 minute– list all the times you have encountered something difficult outside of school; a time when you thought you would never get it, but you eventually did
  - 1 minute– list all of the statements you say in your head when things get hard

Why are your predictions about what students hear in their heads in either situation? Same? Different?

Student Examples – 3rd Graders

#1 Tough times in math

Fractions x, ÷

Counting by 8 Telling time

Base 10 blocks

When the substitute didn't give us all we needed

#2 In our heads - Math

I can never do this. I am not smart enough.

Why, GOD, is _____ so hard?

I don't like math. I will never understand it.

I wish math was never invented.

#3 Difficult time out of school

Irish Dancing

Twister Sports

Getting friends

Getting on a rollercoaster

#4 In our heads

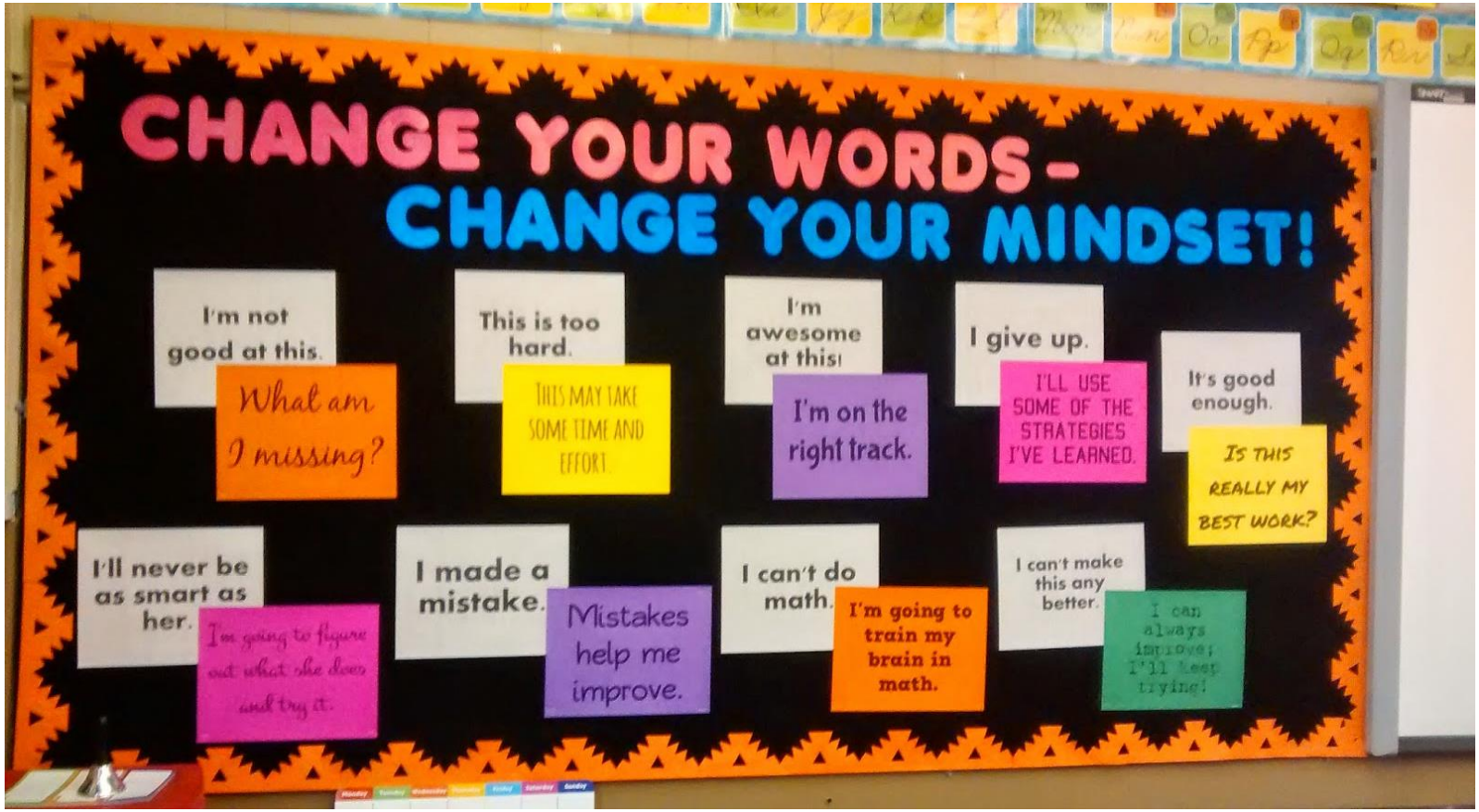
I can do this.

I got this. I'll try my best.

Never give up.

Don't stop believing.

Classroom Activities for Growth Mindset



Making (Embracing) Mistakes

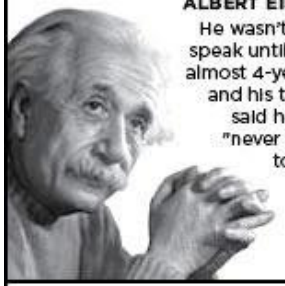
- "My Favorite No"

- Making mistakes and correcting them builds the bridges to advanced learning.

  -Brown, Roediger & McDaniel, 2014, p.7

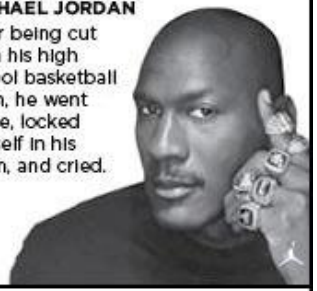
- Be a cheerleader for mistakes

FAMOUS FAILURES



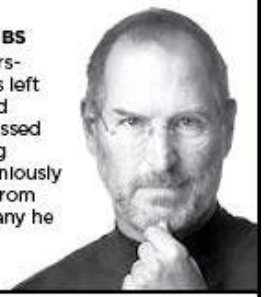
ALBERT EINSTEIN
He wasn't able to speak until he was almost 4-years-old and his teachers said he would "never amount to much"

MICHAEL JORDAN
After being cut from his high school basketball team, he went home, locked himself in his room, and cried.



WALT DISNEY
Fired from a newspaper for "lacking imagination" and "having no original ideas."

STEVE JOBS
At 30-years-old he was left devastated and depressed after being unceremoniously removed from the company he started.



OPRAH WINFREY
Was demoted from her job as a news anchor because she "wasn't fit for television."

THE BEATLES
Rejected by Decca Recording Studios, who said "We don't like their sound—they have no future in show business."

IF YOU'VE NEVER FAILED,
YOU'VE NEVER TRIED ANYTHING NEW

More questions: Maintain cognitive demand

- Encourage students to talk about what they are doing during a math task.
  - How did you approach the problem?
  - What worked? What didn't work?
  - What do you know for sure to be true?
  - What math are you using today that you were unable to use last week? Last month?
  - What are the ideas you still haven't tried?
  - Are there any tools that might help you?

Shopping Trip Task

- Read the Shopping Trip Task
- Discuss: How does this example demonstrate the importance of maintaining high cognitive demand while implementing a task?
- What strategies can we take away from this example?

Tools can make all the difference

- Where do you keep your math manipulatives?
  - Unifix cubes? Base 10 blocks? Clocks? Dice? Rulers? Yard Sticks? Scales? Graph paper? Grid paper?Square/Centimeter tiles?
  - Scissors? Scratch paper? Colored pencils?

- What happens to the cognitive demand when you pull out specific tools for different activities or unit of instruction?

- Why might we want our students to have access to ALL tools ALL the time?

Student Discourse as Formative Assessment

- Carefully select how students present to the group
- Precise vocabulary
- Look for understandings, misconceptions, or gaps in learning
- These conversations and sharing lead to follow up lessons.

Your turn: Another Example

- David's Room

David is getting new carpet for his room. He measures and his room is 9 ft. by 12 ft. He went to the store and carpet was on sale for $15 per square foot.

Step 1: Draw a diagram of his room with the measurements.

Step 2: The area is how much carpet he will need. Draw an array or multiply to find the area of his room.

Step 3: Each square foot is $15.

Use a multiplication strategy to figure our how much it will cost him.

_____ X _____ = _____

(area) (cost of carpet) (total cost)

Your turn: Another Example

- David's Room

David is getting new carpet for his room. He measures and his room is 9 ft. by 12 ft. He went to the store and carpet was on sale for $15 per square foot.

Your turn: Another Example

- David's Room
- Take a look at the original assignment. What do you notice about the task?
  - What changes would you make?
  - What questions would you prepare in advance?
  - What misconceptions might you need to prepare for?
- Take a look at the modified task?
  - How has the cognitive demand changed?
  - What will the teacher need to anticipate to maintain cognitive demand?

Your turn: Another Example

- Carpeting Jillian's Room

Julia's room is 8 ft. wide and 7 ft. long. She found discount carpet that was 5ft. Wide and 13 ft. long. Will she have enough carpet to cover the entire room?

Use the graph paper to help you figure out this problem.

Draw the dimensions of Jillian's room below.

Use the graph paper to CUT out the dimensions of the carpet.

Your turn: Another Example

- Carpeting Jillian's Room

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Be critical, Be intentional

- Don't feel like you have to reinvent the wheel or create every high demand task on your own.
- Be critical of tasks you locate.
- Be intentional in all aspects of the lesson:
  - Planning
  - Implementing
  - Assessing

Thank you!

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