Mathematics Teaching Practices

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representation to deepen understanding of mathematics concepts and procedures and as tools for problems solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Taken from: *Principles to Actions: Ensuring Mathematical Success for All*, an NCTM publication, 2014

Breakfast for All

Source: Adapted from Common Core Georgia Performance Standards Frameworks

Your team has been asked to create the packaging for a new kind of cereal. The manufacturer wants three different sized boxes:

1. A standard sized cereal box
2. A mini sized box that is half as tall, half as wide, and half as deep as the standard size.
3. A super sized box that is three times as tall, three times as wide, and three times as deep as the standard size.

Predict how the volume will change for each box:

Using grid paper, draw a possible design for each box. Label the dimensions and calculate the volume.

Which box do you think would be the best seller? Why do you think so?

Collecting Basketball Cards

Source: Exemplars K-12

Kent and Allie collect basketball cards. Kent has twenty cards and Allie has thirty cards. Kent's mom says that if Kent helps take care of his little brother while she prepares dinner, she will give Kent five cards every Monday. Allie's mom says that if Allie helps fold the laundry, she will give Allie four cards every Monday. Allie tells Kent that she will always have more cards, but Kent isn't sure about that. Will Allie always have more basketball cards than Kent? Show all of your mathematical thinking.

Domain: Number and Operations-Fractions **Standard Code:** 4.NF.2 **Teacher Name:** Krysten

Adapted from: Smith, Margaret Schwan, Victoria Bill, and Elizabeth K. Hughes. "Thinking Through a Lesson Protocol: Successfully Implementing High-Level Tasks." *Mathematics Teaching in the Middle School 14* (October 2008): 132-138.

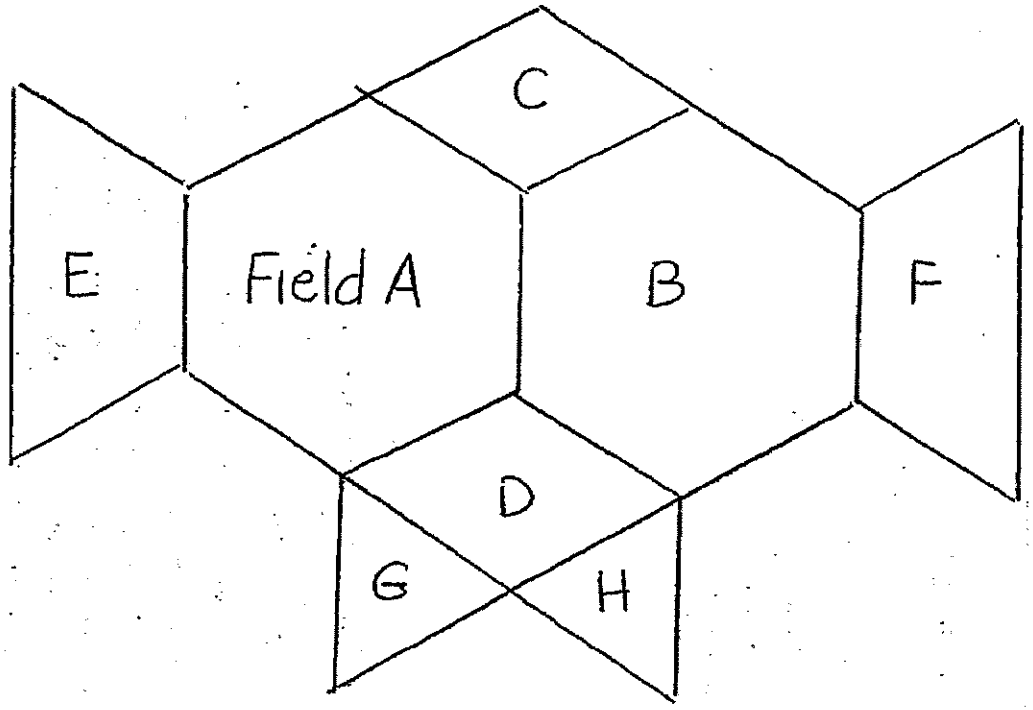
PART 1: SELECTING AND SETTING UP A MATHEMATICAL TASK	
What are your mathematical goals for the lesson? (i.e., what do you want students to know and understand about mathematics as a result of this lesson?)	Students use fractional parts of a whole, properties of shapes, congruency, and computation using fractions and money to determine the fair value and total worth of eight fields.
• What are your expectations for students as they work on and complete this task? • What resources or tools will students have to use in their work that will give them entry into, and help them reason through, the task? • How will the students work—independently, in small groups, or in pairs—to explore this task? • How will students record and report their work?	Students will be expected to participate in the task using appropriate voice levels and everyone will be accountable for the information gathered. Students will use their math journals, pencils, crayons, pattern block manipulatives. Students will work in small groups to explore this task Students will record their work in their math journals and will share them with the class on the document camera. Students will justify their process to the class
How will you introduce students to the activity so as to provide access to *all* students while maintaining the cognitive demands of the task?	Create big construction pieces of farmer Fred's field. Play the song Old MacDonald Had a Farm as you put the pieces of the field on the board. Tell the students you are going to be helping farmer Fred figure out how much each piece of his land is worth.

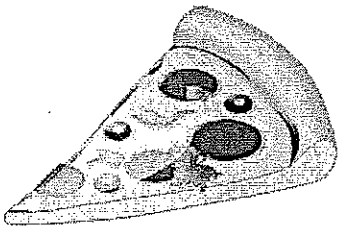
PART 2: SUPPORTING STUDENTS' EXPLORATION OF THE TASK	
As students work independently or in small groups, what questions will you ask to— • help a group get started or make progress on the task? • focus students' thinking on the key mathematical ideas in the task? • assess students' understanding of key mathematical ideas, problem-solving strategies, or the representations? • advance students' understanding of the mathematical ideas?	1. What is going to be your first step in solving this problem? 2. Tell me how you got this answer? 3. Explain the process you used. 4. What else could you try? 5. Can you find another way to solve this problem? 6. Can you explain how you got your answer to someone else? 7. Can you prove your answer?
How will you ensure that students remain engaged in the task? • What assistance will you give or what questions will you ask a student (or group) who becomes quickly frustrated and requests more direction and guidance is solving the task? • What will you do if a student (or group) finishes the task almost immediately? How will you extend the task so as to provide additional challenge?	Reinforce students that they are capable of solving the problem Break the problem apart and look at little pieces of a time What patterns do you see that could help to solve this problem Students will make a list of all the mathematical concepts they used to solve the problem and then will create their own problem as a group to share with the class.

PART 3: SHARING AND DISCUSSING THE TASK	
How will you orchestrate the class discussion so that you accomplish your mathematical goals? • Which solution paths do you want to have shared during the class discussion? In what order will the solutions be presented? Why? • What specific questions will you ask so that students will— 1. make sense of the mathematical ideas that you want them to learn? 2. expand on, debate, and question the solutions being shared? 3. make connections among the different strategies that are presented? 4. look for patterns? 5. begin to form generalizations?	Walk around the room taking notes on what each group is doing and what order you want to share in. Every group will share their findings. Start with the most simple solutions and then move to the most complex answers. After everyone shares their answers, what were the common findings within the groups? How were they different? Are there other ways to solve this problem that were not shared? Did anyone come up with another way to solve the problem, after watching the other groups go?
What will you see or hear that lets you know that *all* students in the class understand the mathematical ideas that you intended for them to learn?	Debrief the task with the class talking about the mathematical concepts that were taught. Make sure the objective for the lesson was reached.

Farmer Fred

Farmer Fred's fields are worth twelve hundred dollars total. The fields are formed with the same properties as your pattern blocks. Each field's value is based on its size. What fraction of the total value is each field worth? How much is each field worth? Show and explain all of your mathematical thinking.





CONSTRUCTING TASK: PIZZAS MADE TO ORDER

Adapted from a Learning Task by Cara Coker, Floyd County, GA

APPROXIMATE TIME: 1-2 class periods

Students will fill pizza orders by representing the ordered ingredients on the appropriate fractional parts of a pizza cut-out.

CONTENT STANDARDS

MGSE3.NF.1 Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into *b* equal parts (unit fraction); understand a fraction $\frac{a}{b}$ as the quantity formed by *a* parts of size $\frac{1}{b}$. *For example,* $\frac{3}{4}$ *means there are three* $\frac{1}{4}$ *parts, so* $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

MGSE3.NF.3 Explain equivalence of fractions through reasoning with visual fraction models. Compare fractions by reasoning about their size.

a. **Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.**
b. **Recognize and generate simple equivalent fractions with denominators of 2, 3, 4, 6, and 8, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.**
c. **Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.** ***Examples: Express 3 in the form*** $3 = \frac{6}{2}$ ***(3 wholes is equal to six halves); recognize that*** $\frac{3}{1} = 3$***; locate*** $\frac{4}{4}$ ***and 1 at the same point of a number line diagram.***
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Before the activity, be sure the children understand the concept of equal parts. Practice with the student's methods to divide various shapes into fractional pieces. Have students practice drawing lines to divide squares, rectangles, triangles, and circles into halves, fourths, eighths.

COMMON MISCONCEPTIONS

Students do not understand there are many fractions less than 1. Students do not understand fractions can be greater than 1.

ESSENTIAL QUESTIONS

- How can I represent fractions of different sizes?
- What relationships can I discover about fractions?
- What is a real-life example of using fractions?

MATERIALS

- Give Me Half! By Stuart J. Murphy (or another book about the concept of fractions).
- Scissors
- Glue or paste
- Crayons
- One large sheet of black paper
- One half sheet of brown paper
- Small pieces of various colored paper including red, white, green, yellow, black
- Pizza Order Directions – One per child

GROUPING

Individual Task

NUMBER TALKS

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students.

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
To assess prior knowledge, brainstorm with students about food that is divided into equal pieces. Possible suggestions may include a chocolate bar, apple pie, pizza, and an orange. Read aloud and discuss, *Give Me Half!* By Stuart J. Murphy (or another book about the concept of fractions).

Part II (SMP 1, 2, 4, 5)

To begin the lesson, give students a half sheet of brown paper. Instruct them to draw and cut out a circle from the brown paper. Then give each child a Pizza Order. Instruct the students to use their pencil to divide their circles into the fractional part used in the Pizza Order (fourths or eights). Then have the students trace over their pencil lines with a dark crayon. Next, give students small sheets of the colored paper (red, white, green, yellow, black). Instruct students to cut pieces of the colored paper to represent the pizza toppings. The toppings should be glued onto the appropriate number of pizza slices.

After the toppings have been successfully glued to the brown circle, give each student a sheet of black construction paper. Have the students glue their pizzas and Pizza Order Directions to the paper.

FORMATIVE ASSESSMENT QUESTIONS

- What fraction of your pizza is covered with peppers?
- What topping covers most of your pizza?
- Are black olives covering more or less than half your pizza?
- How did you divide your pizza into equal parts?
- How many equal parts did you need? How did you know?
- If your whole pizza was divided into fourths, how many slices did you cover with toppings? How would you write this as equivalent fractions? (4/4 = 1)
- If your pizza is covered with 1/8 mushrooms and 3/8 green peppers, does it have more mushrooms or green peppers? How do you know? (Encourage students to explain in terms of the pizza size and by comparing numerators in the fraction.)
- Some of you covered 4/8 of your pizzas with pepperoni. Can you name equivalent fractions for 4/8?
- Were any pizzas covered with ½ cheese? Why did your Pizza Order ask for 2/4 cheese?
- Do you see any other examples of equivalent fractions on the pizzas?

DIFFERENTIATION

Extension

- Have students create additional pizzas using more challenging fractional parts such as thirds, sixths, tenths. Increase the number of toppings. Have some sections contain more than one topping.

Intervention

- Provide ready-cut circles and if necessary, draw dotted lines for students to trace as they divide their pizzas into fractional parts. Have students complete Pizza Orders using fractions containing only common denominators.

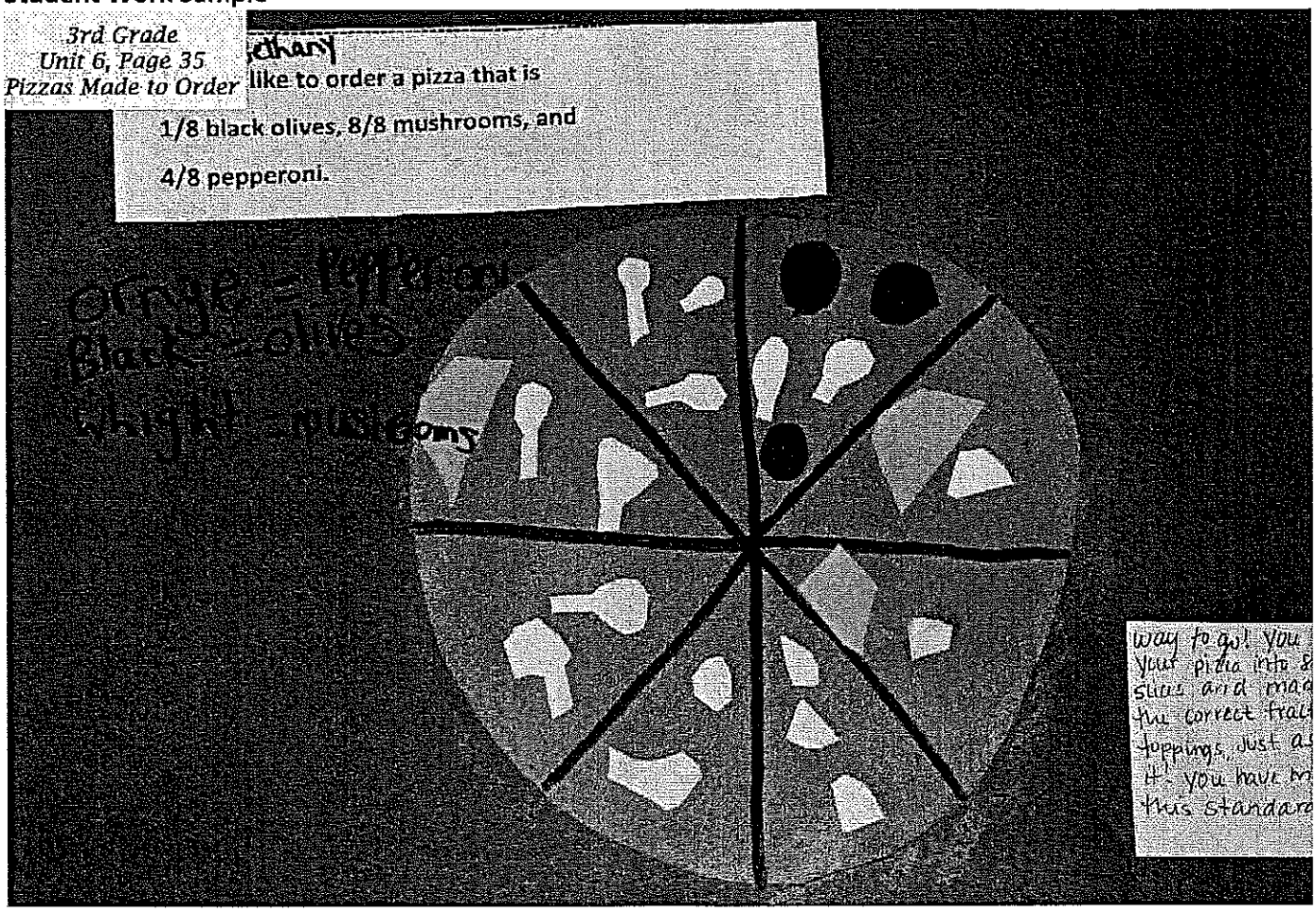
TECHNOLOGY RESOURCES

http://mrnussbaum.com/pizza_game/index.html

http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/fractions/index.htm

http://www.primarygames.com/fractions/2a.htm

Student Work Sample



PIZZAS MADE TO ORDER: PIZZA ORDER DIRECTIONS

Adapted from a lesson by Cara Coker, Floyd County, GA

I would like to order a pizza that is 1/8 green peppers, 8/8 pepperoni, and 3/8 mushrooms.

I would like to order a pizza that is
$\frac{1}{4}$ mushrooms, 2/4 cheese, and $\frac{1}{4}$ pepperoni.

I would like to order a pizza that is
1/8 black olives, 8/8 mushrooms, and
4/8 pepperoni.

I would like to order a pizza that is
$\frac{1}{4}$ mushrooms, $\frac{1}{4}$ black olives, and
$\frac{1}{2}$ pepperoni.

I would like to order a pizza that is
$\frac{1}{4}$ cheese, $\frac{1}{4}$ black olives, $\frac{1}{4}$ pepperoni, and
$\frac{1}{4}$ green peppers.