

WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET
WISCONSIN MATHEMATICS COUNCIL

March 6 – 10, 2017

Problem Set #1

Score:
(For Scorer's Use Only)

Name: _____

Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

For this first problem set, calculators are not allowed. They may be used for the remainder of the meet only, starting with Problem Set #2.

Answers

1. (1 point)

Solve for y : $3^{y+1} = 27^y$

2. (3 points)

Let A and B be acute angles where $\tan A = \frac{1}{7}$ and $\sin B = \frac{1}{\sqrt{10}}$. Find the radian measure of $A + 2B$.

3. (5 points)

If $b = \log_5 x$, list all values of x in exact form that satisfy the equation $\log_b(\log_5 x^3) = 3$.

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Problem Set #2

Score:
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Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

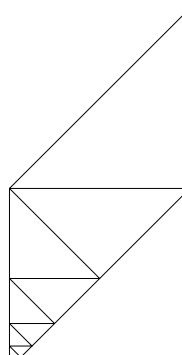
On Twitter, you can type up to 140 characters in one message. If you start typing the natural numbers with no spaces between them (1234567891011121314...), what is the last complete 2-digit natural number you can type?

2. (3 points)

A deck of cards comprises 6 red cards, 5 yellow cards, and 4 blue cards. If four cards are randomly selected without replacement, what is the probability that there is at least one card of each color?

3. (5 points)

This figure, made up of ever smaller isosceles right triangles, continues indefinitely. If the area of this figure equals its perimeter, find this common value.



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Problem Set #3

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Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

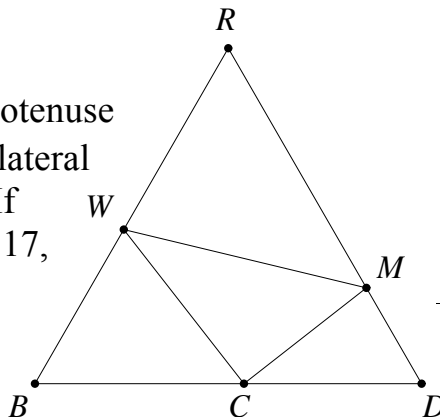
A data set consists of 2017 consecutive integers.
If the sum of all of the numbers in this data set is
24,204, then what is the median of the data set?

2. (3 points)

Find the sum of all positive integers n such that
 $2017 + n^2$ will be a perfect square.

3. (5 points)

Right triangle WMC (hypotenuse \overline{WM}) is inscribed in equilateral triangle BRD as shown. If $BC = 20$ and $BW = CD = 17$, compute MD .



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Problem Set #4

Score:
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Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point)

Ten cards numbered 1 through 10 are placed in a hat. If 4 cards are drawn without replacement, what is the probability that they are drawn in increasing order? _____

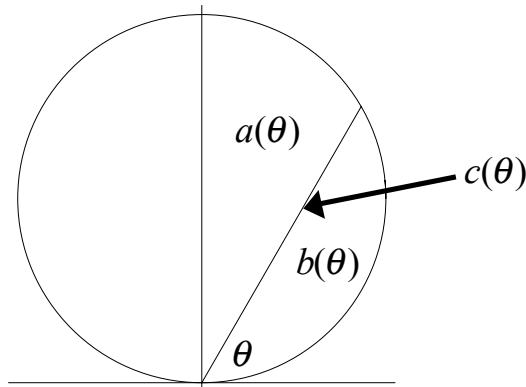
2. (3 points)

Consider the chord $c(\theta)$, which divides the right semicircle into two regions having areas $a(\theta)$ and $b(\theta)$.

When $\theta = \frac{\pi}{6}$ radians, $c(\theta) = 2$.

For $0 \leq \theta \leq \frac{\pi}{2}$, find the

maximum value of the product $a(\theta)b(\theta)$. _____



3. (5 points)

If $\frac{1+2r+3r^2+4r^3+\dots}{1+r+r^2+r^3+\dots} = \frac{7}{11}$, find r . _____

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Team Problem Set (Page 1)

Score: _____
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Team: _____

Captain: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

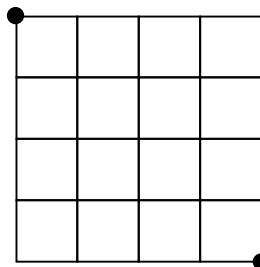
Answers

1. (10 points)

Determine all pairs of integers (x, y) that satisfy the equation $6x^2 - 2xy + 5y - 7x = 12$.

2. (10 points)

Starting from the top left corner of a 4×4 grid and moving either right or down at each step, how many possible routes are there to the lower right hand corner?



3. (10 points)

Two circles of radii 20 and 17 are externally tangent. Compute the sine of the angle formed by their common external tangents in exact form.

Team Problem Set (Page 2)

4. (10 points)

Find the number of ordered pairs (a, b) such that $(a + bi)^{2017} = a - bi$, where a and b are real numbers.

5. (10 points)

On Twitter, a tweet is limited to 140 characters. Let's define a squawk to be a message limited to 1400 characters. If I start typing the natural numbers into my squawk starting with 1 and putting a space after each number (1 2 3 4 5 6 7 8 9 10 11 12 13 14 and so on), what is the last complete natural number in this list that I can type into my squawk?

6. (10 points)

In a televised game show, Bob hopes to win a prize that lies behind one of ten doors labeled "1" through "10."

Bob's preliminary guess is door "3," whereupon the game show host (Monty) opens four of the remaining nine doors, "1," "2," "8," and "10," revealing four empty rooms (no prize). (Monty knows where the prize is located and only opens doors with no prize.)

At this point, Monty asks Bob whether he wants to stick with his preliminary choice of "3" or switch to one of the five remaining unopened doors "4," "5," "6," "7," or "9." Suppose Bob decides to switch to one of these other doors ("7," for example) for his final guess. If he makes the switch, what is the probability that he wins the prize?
