WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET
WISCONSIN MATHEMATICS COUNCIL
March 1 – 5, 2021

Problem Set #1 – The Disco '70s Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point) – from 1972 (our very first contest)

   From 6 oz. of white paint (container A), 1 oz. is put into 6 oz. of red paint (container B). 1 oz. of the white-red mixture in container B is then put into container A. Which resulting mixture, container A or container B, is more diluted from its original state?

   _____

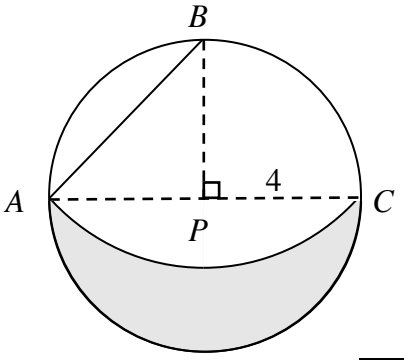
2. (3 points) – from 1974

   A turtle and a rabbit are team members in a 96-foot relay race, each running half the distance. If the turtle runs the first half at an average rate of 4 feet per second, how fast must the rabbit run the second half of the race so that together they average 6 feet per second for the entire race?

   _____ ft/sec

3. (5 points) – from 1979

   In the figure to the right, the radius of circle P is four units and the top border of the shaded region is a circular arc with radius $\overline{AB}$ and center at B. $\overline{AC}$ is a diameter of circle P. Find the area of the shaded region.



   _____ units^2

WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET

WISCONSIN MATHEMATICS COUNCIL

March 1 – 5, 2021

Problem Set #2 – The Awesome '80s Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point) – from 1983

   The sum of three sides of a rectangle is 8 units. What is the greatest possible perimeter of the rectangle if its area is 6 square units?

   _____ units

2. (3 points) – from 1980

   Team A and team B are playing a series of games. If the odds for either team to win any given game are even, and team A must win two games or team B must win three games to win the series, what is the probability that team A will win the series?

   _____

3. (5 points) – from 1985

   John can do a job in 12 days if Jim helps him for 7 of those days. But Jim takes 14 days to do the same job if John helps him for 7 of those days. If John and Jim work at steady rates, how many days would it take John alone to do the job?

   _____

WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET
WISCONSIN MATHEMATICS COUNCIL
March 1 – 5, 2021

Problem Set #3 – The Dot-Com '90s Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

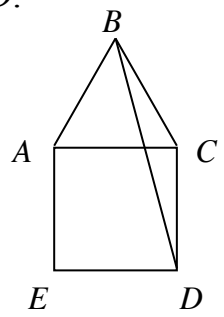
Answers

1. (1 point) – from 1997

   If 1, 2, and 4 are three of the digits of the four-digit number N, and if N is divisible by 4, what is the greatest possible value of N? _____

2. (3 points) – from 1991

   Equilateral triangle ABC rests on top of square $ACDE$. Find the measure of $\angle ABD$. _____ °



3. (5 points) – from 1994

   McDonald's sells Chicken McNuggets in boxes of 6, 9, or 20 nuggets. Call "N" a McNugget number if it is possible to order boxes of McNuggets such that the order contains exactly N McNuggets. [For example, 12 and 15 are McNugget numbers, but 10 and 11 are not.] What is the largest number that is not a McNugget number? _____

WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET
WISCONSIN MATHEMATICS COUNCIL
March 1 – 5, 2021

Problem Set #4 – The Millennial '00s Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point) – from 2009

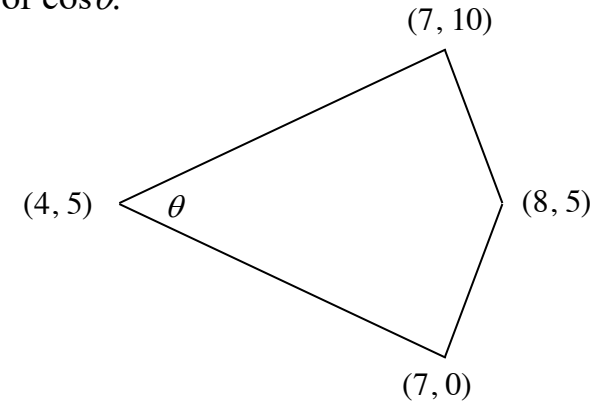
   If $\sin x + \cos x = \sqrt{2}$, then $\sin x \cos x = ?$ _____

2. (3 points) – from 2009

   At the grocery store, Mary bought 2 apples, 3 grapefruit, and 5 peaches, and she paid \$9 for them. Chris bought an apple, 2 grapefruit, and a peach, paying \$4 for them. Deb bought 5 apples, 4 grapefruit, and 3 peaches, for a cost of \$10. How much does a single grapefruit cost? _____

3. (5 points) – from 2001

   In the quadrilateral shown below, find the **exact value** of $\cos\theta$. _____



WISCONSIN HIGH SCHOOL STATE MATHEMATICS MEET

WISCONSIN MATHEMATICS COUNCIL

March 1 – 5, 2021

Problem Set #5 – The Recent '10s

Score: (For Scorer's Use Only)

Name: _____ Team: _____

[Reduce all common fractions. Simplify and rationalize denominators. Unless otherwise specified, a decimal approximation will **not** be accepted. When allowed, round decimal approximations to **3** decimal places. **No rounding should be done except on the final answer.**]

Answers

1. (1 point) – from 2018

   If $\log_b 2 = x$ and $\log_b 5 = y$, write $\log_b 20^{18}$ in terms of x and y.

   _____

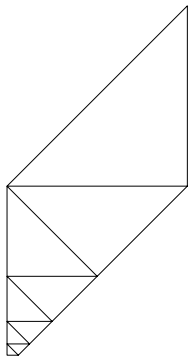
2. (3 points) – from 2011

   Pirates are a notoriously lazy lot. To motivate them, the Captain declares that on days that they work capturing merchant vessels, they earn 7 pieces of gold. On days that they lag about on the deck and don't work, they will be charged 5 pieces of gold. After 365 days of plundering, each pirate has earned a net total of only 11 pieces of gold. How many days did they work capturing merchant vessels?

   _____

3. (5 points) – from 2017

   This figure, made up of ever smaller isosceles right triangles, continues indefinitely. If the area of this figure equals its perimeter, find this common value.

   _____

Problem Set #5 (Page 2)

50^{th} Anniversary Problem (not appearing on any past problem sets)

4. (5 points) – new problem for 2021

An unfair coin has a $\frac{20}{21}$ probability of landing heads side up when flipped. If this coin is flipped 50 times, what is the probability that the total number of heads is even?
You may express your answer as a decimal approximation or as an exact numerical expression.

_____