Orange Problem

Synopsis
In this problem students will rely on several problem solving strategies to develop their algebraic thinking. Students could act out the problem, draw pictures, develop table, create graphs, and use symbolic representations. Additionally, students will explore patterns and develop recursive and explicit formulas. Recursive formulas involve predicting values in a table by looking at previous values and search for patterns. Explicit formulas involve substituting an input from the table to determine an output that is used for prediction.

Connections to the NCTM Standards
Algebra
- represent and analyze patterns and functions, using words, tables, and graphs
- represent the idea of a variable as an unknown quantity using a letter or a symbol
- express mathematical relationships using equations
- model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions

Problem Solving
- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Materials Needed
- Oranges or Manipulatives that represent oranges
- Calculators
- Orange Problem Handout
- Graph paper and straightedge
- Paper and pencil

Description of the Activity
Begin the lesson by distributing manipulatives representing oranges to each pair of students. Distribute a copy of the Orange Problem to the students. After students have received the handout, use a Think-Pair-Share strategy to get students to brainstorm strategies. Ask students to think individually about problem strategies. The students should take 5-10 minutes to think individually; then each student should pair up with a partner to discuss common findings. Finally, the entire class should engage in a conversation to share strategies. The teacher should elicit varying strategies such as students' drawing, tables, acting out, and others. After the strategies have been shared,

ask students to solve the problem for 4, 5, 10, and n customers. Again, use Think-Share-Pair and record students' solutions on a table similar to the table below.

Number of Customers	Number of Oranges Sold
1	1
2	3
3	7
4	15
5	31
n	$2^n - 1$

Notice the pattern in the number of oranges sold. Students may notice the recursive pattern of +2, +4, +8, and +16. Students can use the recursive pattern to develop and explicit formula.

Orange Problem

A grocer was asked how many oranges he had sold that day. He replied:
> "My first customer said I'll buy half your oranges and half an orange more."
>
> He then said, "My second customer said the same thing… I'll buy half your oranges and half an orange more."
>
> Then he stated, "My third customers said the same thing... I'll buy half your oranges and half an orange more."
>
> Finally, he stated, "When I had filled all three orders I was sold out and I did not have to cut a single orange all day."

How many oranges had the grocer sold in all?

What if there were four customers? Five customers? Ten customers? Any number of customers?

"THE KIDS"

Thomas
Thomas defines himself as someone who is "not good" at math. He has problems focusing and paying attention in class but does not know how to help himself. As a result, Thomas gets very little from his daily classroom mathematics instruction. Thomas often confuses information on a test or worksheet, by combining pieces from one problem with information from another problem. He enjoys working on problems that can be solved in a sequential manner and is very adept at using a calculator. Thomas enjoys working with other students and is always willing to contribute in small group discussions

Heather
Heather has trouble expressing herself in class. She becomes visibly distressed when asked to explain her answers. Heathers verbal responses are unorganized, use endings incorrectly, and lack appropriate syntax. Heather does not appear to listen in class; perhaps because she does not fully understand all that is asked of her. Heather loves math and her teacher believe that she understands more than she is able to apply. She works well in a cooperative learning environment. She makes friends easily. However, when working independently, she has a hard time getting started on assignments without assistance. Heather enjoys sports and is a terrific tennis player.

Eric
Eric is a very quiet student. He comes into class quietly and complies with requests without fussing. He turns in all of his work but many problems are incorrect or incomplete. Eric never asks questions, avoids eye contact with the teacher but he relates comfortably with two other students in class. He is often absent on test and quiz days. Eric can recite math facts and use them proficiently for computation problems. His weaknesses are primarily in the area of solving contextual problems. Eric loves animals. He spends a great deal of time reading information on different kinds of animals. In addition, he loves to draw; usually animals.

Kevin
Kevin struggles with reading and feels overwhelmed when there is a lot of text on a page. He often gets confused by the wording of directions and therefore is unsure what he is being asked to do. Embarrassed by these difficulties, Kevin is hesitant to ask the teacher or his classmates for help. He is more comfortable when things are presented visually. Often, he draws pictures as a way to figure out the solutions to problems.

Ingrid
Ingrid gets easily distracted when the teacher is giving directions and has trouble sitting still during class discussions. Sometimes, she tunes out of the mathematics lesson and tries to sneak reading a book in her desk. Although she does not listen well to her classmates, she is quite articulate and likes to talk about her own ideas, particularly when she works in small groups. Ingrid tends to rush through mathematics problems, leaving out important parts and making careless errors.

Daniel
Daniel's desk is a mess. He can spend the entire mathematics lesson looking for his mathematics book or his homework. When he has a multi-step problem to solve, he tends to lose track of the steps, gets confused, and not finish in time. Danny often manages to get by because he has an excellent memory for mathematics facts and vocabulary. He sometimes misses social cues so his classmates do not like to work with him on small-group activities.

Sara
Sara has difficulty making connections with prior lessons, so each problem looks new to her. Often, she looks blankly at the page and waits for the teacher to help her. When Sara knows what to do, she works slowly and carefully. Her papers are neat and well-organized. In class discussions, she is eager to participate but tends to talk about the steps she used to solve a problem without explaining why she used those steps.

Vignette 1. Calvin (Adapted from Berry, 2008)

Calvin is a sixth grade Black boy who considers himself to be smart with a little "swagger." He attends school in an urban school division located in a southeastern state. As an elementary school student, Calvin earned the highest level of achievement on the third, fourth, and fifth grade state standardized mathematics tests. On all objective measures in mathematics, Calvin has performed well and in most cases has excelled. In addition, he has earned good grades in mathematics by earning primarily A's with an occasional B. Calvin stated that mathematics is his favorite subject and that mathematics comes naturally to him and is easy. He loves challenging mathematics problems and mathematics puzzles.

Calvin's mother acknowledges that her son a "busy body" and is in need of a variety of stimulation in order to prevent boredom. She also stated that Calvin needs to feel that his teachers are interested and cares about him in order for him to be productive in class. Both Calvin and his mother admit that he can be a handful in class. Occasionally, he speaks out or is not in his seat at the appropriate time. His behavior is not always that of a model student; however, they believe his behavior is well within acceptable classroom norms.

At the end of fifth grade, Calvin was excited about going to middle school. At that time, teachers identified students to take a mathematics placement test to gain entry into an upper-level pre-algebra mathematics course for sixth graders. Calvin was upset because he was not selected and there were students selected to take the test who he considered were not as "good at math." Calvin's mother inquired about the criteria for selection of taking the placement test and discovered that Calvin met all criteria except one, teacher recommendation. Calvin's fifth grade teacher indicated that although Calvin scored well on assessments, his behavior and his inability to sit still would not make him a good candidate for pre-algebra in sixth grade. In a conference with the sixth-grade guidance counselor, Calvin's mother inquired about placement in pre-algebra. The guidance counselor responded that she would not want to place Calvin in a class he would not do well. Calvin's mother felt that the counselor did not consider Calvin's previous mathematics performance and focused on other things. The principal at the middle school evaluated Calvin's situation and argued that pre-algebra is a rigorous course for sixth grade students and only disciplined students are capable of passing this course. Even though Calvin had performed well in mathematics throughout his schooling, school personnel focused their attention on behavior rather than academics when evaluating his potential. When the sixth grade school year began, the pre-algebra class had no Black male students.

Calvin's school district is concerned about the achievement gap. In fact the school division has a goal statement focused on the achievement gap stating "it seeks to understand the causes of this gap in order to devise solutions to reverse it." Calvin's story raises questions about beliefs that school districts hold for Black boys. Fortunately, Calvin had a persistent mother who advocated for her son and challenged the school division and Calvin gained entry into the pre-algebra class the second week of the new school year. Unfortunately, Calvin's story is not unique; Black boys are often confronted with lowered expectations even when they have shown that they are capable of achieving. If school districts are serious about understanding the needs of all students, then they should critically assess possible structural and systemic factors that contribute to access issues that impact Black boys.

Vignette 2: Caroline and Craig (Adapted from Chval & Davis, 2009)

Caroline
Caroline is a gifted seventh grader who has access to challenging mathematics in both her gifted pull-out program and in her mathematics class. Caroline participates in the pull-out program two days a week with other gifted students. Thus, she and the other students are with their teacher only three days a week. However, her teacher recognizes the importance of differentiating instruction for her students every day. She realizes that she must give careful consideration to this instruction because gifted children require different and more flexible educational experiences. As a result, Caroline's teacher provides thought-provoking problems and structures them in ways that provide multiple entry points for the whole class. She also encourages her students to demonstrate what they know during small-group and whole-group discussions, creating a safe and respectful environment where all students can solve problems in different ways. This classroom environment makes Caroline truly enjoy her mathematics class because she feels respected, engaged, challenged, and creative. All these elements will allow her to excel in mathematics.

Craig
Craig, a gifted seventh- grade middle school student, is not engaged during his mathematics lessons. The content is not difficult for Craig, and his participation is not encouraged. For example, his teacher often says, "Craig, I know you know the answer. I want to see if anyone else knows." This statement and similar comments have taught Craig not to raise his hand in class. In addition, his teacher frequently tells him that he cannot use his mathematics knowledge to reach an answer because some of the other students have not yet learned it. For example, when his class was studying circles, Craig was told not to use pi or his algebra skills to calculate area and circumference. This and similar situations have frustrated Craig. As a result, he has learned not to initiate questions or alternative approaches to solving problems. Later, during the school year, Craig approached his teacher to request some challenging problems to work on independently during class. Although the teacher took additional time to find mathematics problems that would challenge Craig, he asked that Craig solve them outside of class. This gesture helped challenge Craig but did not improve his classroom experience. Craig disliked his middle school mathematics class because he felt that he was not respected, engaged, or challenged. He was also prohibited from solving problems using different methods than those used by his peers. Craig s role in his mathematics class- room had been reduced to observing or tutoring his classmates, rather than learning.