USING MANIPULATIVES TO DEVELOP CONCEPTUAL UNDERSTANDING IN A MIDDLE SCHOOL MATH CLASSROOM

Lynn Schaal, New London Middle School, New London, Wisconsin

Activities Discussed in this Article can be found at:
https://drive.google.com/drive/folders/1CEBWrtns72gSB6F4t-DoqWwjR1Qxbnrg?usp=sharing

Early in my career as a middle school math teacher, I struggled understanding why students did not just remember formulas or methods needed to solve problems. I quickly realized they did not remember the proper method to use because they did not understand what that process meant or why they were doing what they were doing to use that process. Through my classroom observations and experiences, it appeared the math was not at all meaningful to most students and seemed to be magnified with my struggling math students. So I began to develop methods to justify the process. It was after working closely with other math educators that I began to ask myself and my other math co-teachers at the middle level why we were not using methodology they were using in the elementary math classrooms. It was at that point I began my quest to find manipulatives for my math classroom.

Why Use Manipulatives?

Using manipulatives as part of my math instruction allowed students to understand the abstract concepts in the secondary standards. The use of formulas or algorithms to solve "naked" problems did not carry over in my students' ability to use those methods when problem solving using rich tasks. After attending professional development and looking at the instructional practices of colleagues at the elementary level, I was intrigued by the use of various manipulatives and strategies to develop a deeper understanding of the math concepts. After implementing activities using these strategies and activities, I found that students were able to explore mathematics tactilely and learn mathematical concepts with a deeper understanding.

Research shows math classrooms need to be places where students are able to construct and discover mathematical concepts in a hands-on, meaningful, and connected way (NCREL, 1995). Manipulatives are a great tool to allow for this because they allow students to create concrete representations of abstract concepts as they form their own mathematical understandings (Burns, N.D).

There are a few manipulatives I use more often than others and I am going to share some of those with you along with activities I use. All information is available using the link in the beginning of this article.

Cuisenaire Rods

Cuisenaire Rods are a collection of rectangular rods (Figure 1), each of a different color and size. The smallest rod is one centimeter long and the longest rod is 10 centimeters long. I have found that Cuisenaire Rods are a highly effective way for students to conceptually understand fractional relationships. I utilize Cuisenaire Rods most when teaching rational numbers to review and model operations with fractions. To begin the lesson, I inquire how to add fractions. Students rarely are able to tell me why getting a common denominator is necessary; most of the time they say "just because".

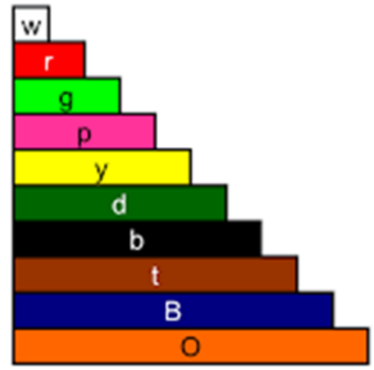

Figure 1

Figure 2a

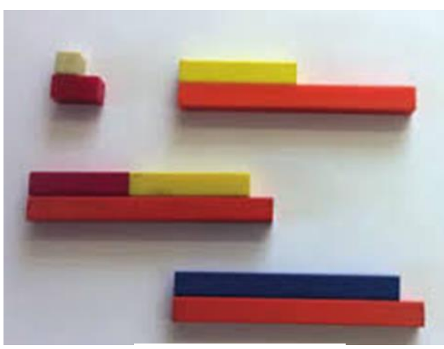

Figure 2b

I use modeling to show the importance of making equivalent fractions when comparing. Questioning techniques and the vocabulary I use are a very important part of this modeling. I use terminology like "doubling, copying, and reproducing" to help students see the importance of creating an equivalent fraction with the "same size whole". Once I have modeled several problems using Cuisenaire Rods on Smart Notebook while they are using the actual manipulatives, I then model by using graph paper to sketch the visual representation. "To engage students in productive visual thinking, they should be asked, at regular intervals, how they see mathematical ideas, and to draw what they see. Drawing mathematical ideas helps mathematics users of any level, including mathematicians, to formulate ideas and develop understandings."(Boaler, 2016) See figure 2a and 2b.

This visual representation is also beneficial when students are unable to have access to a set of rods. After several problems are completed, I have them add the numerical representation next to the visual to assist them in moving to the abstract. I think this helps them see the connection to the symbols and representations of the fraction as a number. I then repeat this process through adding, subtracting, multiplying, and dividing fractions. Each day is spent with one operation, and at the end of class I have them share their work and explain their thinking. (See "Cuisenaire Rods Folder" for activities and links to video clips in the linked folder)

This process does take a week to complete in an 88-minute block. At the end of the week, the students have a deeper understanding of fractions.

Algebra Tiles

Algebra Tiles are one of my favorite manipulatives to use in my math class. Algebra Tiles are square and rectangular tiles that can be used to represent integers, algebraic expressions, and equations (Figure 3). These tiles visually and concretely show the abstract. Students are able to manipulate tiles to: represent positive and negative values, regroup variables and constants, solve equations, substitute in equivalent expressions, and expand and factor expressions. The utilization of tiles in my math classroom allows my students to develop a better understanding of the procedural process of solving problems using algebra.

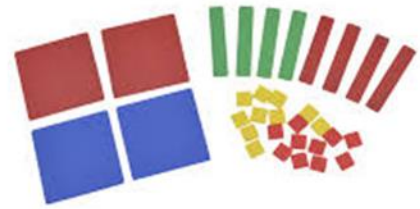

Figure 3

I begin using Algebra Tiles to conceptually show how to add, subtract, multiply and divide integers. I use the small square tiles to represent a +1 (yellow side) and -1(red side). I stress the yellow and red squares are additive inverses of each other, so they have a sum of zero when added together. That concept is important to understand when students are working with algebraic expressions, equations, and inequalities. This pairing of a positive and negative is referred to as a "zero pair". An understanding of zero pairs provides a foundation for my students during the introduction of solving Algebraic Equations; more specifically, the conceptual understanding of adding the opposite to get the variable alone. Zero Pairs do not only refer to integers, but they can be used when using the x or –x (green and red rectangles) or the x^2 or the - x^2 when referring to the blue, or red, large square.

Just like when using Cuisenaire Rods, modeling when using algebra tiles is essential. I begin each by modeling. I model several problems using algebra tiles on Smart Notebook while they are using the actual manipulatives. I then model using sketches of the visual representation. I require them to sketch practice problems because I want them to have an option of drawing the diagram when they are unable to have access to a set of tiles (Figure 4). Once several problems are completed, I have them add the numerical representation next to the visual to assist them in moving to the abstract as well as substituting the solution in the abstract to check their solution. This justifying of their answer is initially very difficult for students. The process of writing the abstract with the manipulatives helps students see the connection to the symbols and representations. I then repeat this process through addition, subtraction, multiplication, division, and two-step equations. Each begins with simple equations and moves into situations where they have the single variable on the right and left side of the equation. They may need to simplify both numbers and variables, as well as variables with a negative coefficient. Students work combining like terms by combining the same shape tiles and making zero pairs. The Algebra Tiles create a clear visual of the necessity to get the variable alone (rectangles) and the need to determine the zero pairs necessary to complete the task.

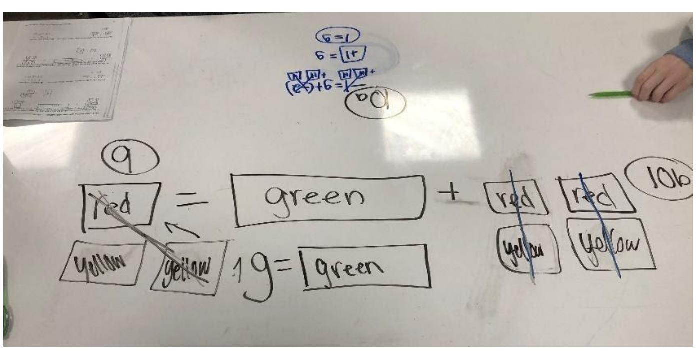

Figure 4

For example, in the picture on the right, students combine the like terms (squares that represent an integer) then determine that they need to get 'g' alone. The green rectangle is currently being added to two red squares (-2). So this student added the opposite of two red squares, two yellow squares (2), to make a zero pair. To ensure that the equation maintains equality they did the same to the other side of the equation. This then resulted in g = 4 yellow squares (4). Then students use the numerical representation to solve the equation and are able to see the meaning behind those numerical representations. The final step in solving this problem is to check your work. Students then substitute the variable value into the equation and determine if they were correct. (See "Algebra Tiles Folder" for all activities located in the linked folder referenced at the top of the article)

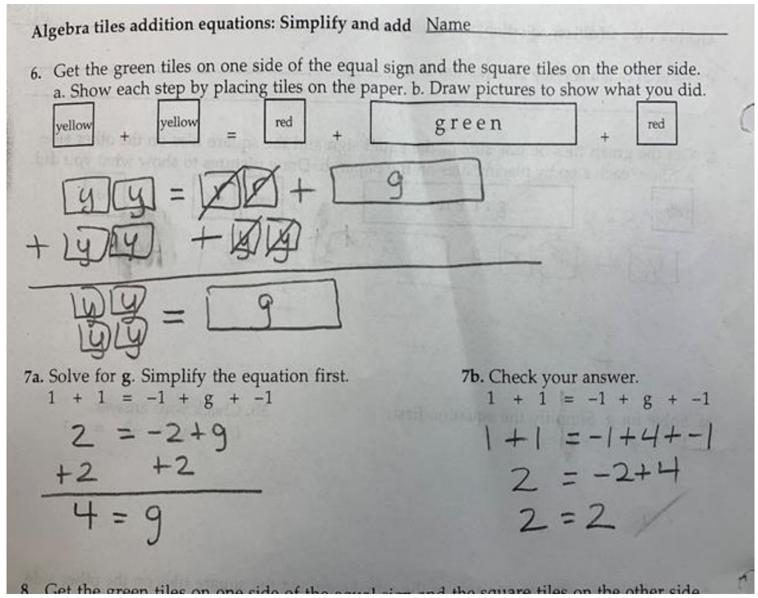

Figure 5

In my math classes, this process has resulted in a deeper understanding of the algebraic process of solving equations and why the process works. My students have developed a clearer vision of the steps they need to solve a problem algebraically.

AngLegs

AngLegs enable students to study polygons, perimeter, area, angle measurement, side lengths, and more. An AngLeg set includes 72 snap-together pieces (12 each of six different lengths) and two Snap-On View Thru® protractors (Figure 6.1 and 6.2). The use of AngLegs allows students to explore and construct triangles using the assigned colored lengths. Students are able to determine the angle measure and classify the triangle by its sides and angles. Through this exploration students develop "rules" regarding the Triangle's angle measures with relation to the Triangle's side lengths.

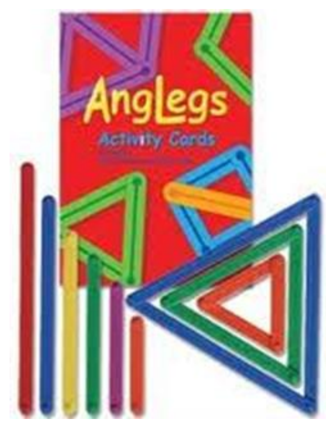

Figure 6.1

In my math classes I utilize the AngLegs to provide exploration of the different types of angles that make up a triangle. Students work through an activity, Triangle Task, to construct triangles of different angle measures and side lengths. The AngLegs are color coded by length and students have to use the protractor to measure the angle that results from the various combinations. Students are then able to physically see and prove the relationship between the side lengths and the measures of the angles. This activity also reinforces that the angles of a triangle measure 180°. This beginning introduction of triangle properties becomes very useful when introducing the Pythagorean Theorem, interior and exterior angle relationships, and similar triangles. (See “Triangle Activity with AngLegs” in the linked folder)

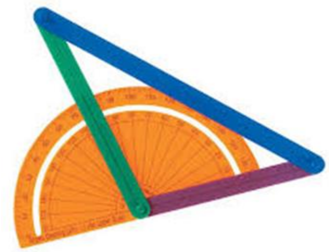

Figure 6.2

Manipulatives can also work at the high school level to reinforce concepts and to demonstrate abstract concepts to students in a concrete manner in math and science classes. One of the high school teachers in my district used AngLegs in his physics class with students who struggle with concepts like analyzing forces that act on objects that are sitting on a ramp, as shown in figure 7. The biggest challenge for students is realizing that the angle between the vertical force of gravity (Fg) and the normal force (Fn) perpendicular to the ramp is the same as the angle of the ramp. AngLegs give students a way to physically prove it.

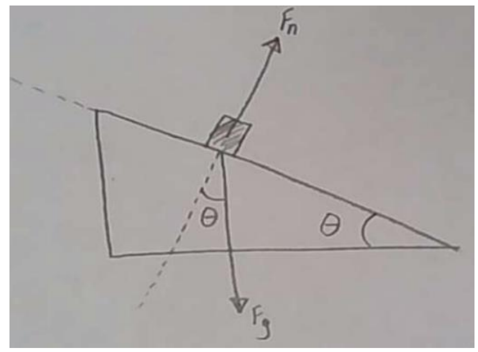

Figure 7

Exploring Surface Area using 3D Solids –Nets

When teaching area and volume, I do not rely on a formula. In my experience students do not remember them when given a formula to use without understanding why that formula works. When beginning my unit in Geometry, students explore the concept of area. We review the elementary concept of Area Models using squares and rectangles, and we use explorations to reinforce what area means. From there we look at the parallelogram, rhombus, and triangle to see how we can decompose, and rearrange, those shapes to make them look like a rectangle or square. Now I know many will say, students should know the formula for the area of a square, rectangle, parallelogram, rhombus, and triangle, but this quick 20-minute activity reinforces and reviews prior knowledge as well as creating an opportunity to assist struggling students by re-teaching.   This decomposing of parallelograms and rhombi is really beneficial when looking at other shapes like trapezoids, composite 2D shapes and surfaces of 3D solids. The materials I use to move through this process are simply different size shapes that I have cut out of construction paper, and square tiles and graph paper to assist with the representation of the area.

Surface Area and Volume are very important concepts in seventh grade math. I have found that using 3D Solids with Removable Nets, as seen in Figure 8, allows students to investigate how to calculate surface area without having to rely on the formula. The formula is efficient in calculating surface area, and that is where I ultimately want them to end up.

Figure 8

However, I have found that the formula is much more meaningful to my students if they develop it based on their exploration of the nets.

In my classroom we begin by investigating 3D solids. Students have an opportunity to "play" with the solids and make observations about the different shapes that make up the solid. Even in seventh grade, I believe this opportunity to just play and check out the various solids gives students a chance to really analyze the solids and compare and contrast 3D Figures. I ask them to try to name the solid. Some may have the correct name, or know the difference between the prisms and pyramids, but not necessarily know the proper name. This quick activity provides an opportunity for students to activate their prior knowledge as well as review the vocabulary associated with three dimensional figures. (See "Playing with Solids" activity in the linked folder)

When working on surface area, I use those same solids, but have them work in small groups to analyze the different solids and their nets. Students are not given the formulas for surface area. They investigate and explore what surface area is and how they can calculate it using the net. I ask them to draw the 3D shape as well as the net. They measure the net using a unit of measure of their choice. They determine very quickly the smaller the measure the more accurate they can be. Students name the figure with the proper name and identify the shapes of the faces and bases. From this investigation students are able to determine and fully understand a formula to calculate surface area. This process also allows them to be able to problem solve when they cannot remember the formula through decomposing the figure into 2D shapes. (See "Discovering Surface Area" activity in the linked folder)

When teaching volume I use Snap or Unifix Cubes for student explorations (Figure 9). I have students explore the relationship between area and volume. Students know the formula for volume of a rectangular prism but often do not use it correctly. Using the cubes allows students to investigate and determine what the formula for volume of a rectangular prism means conceptually. This method also connects nicely for them to see the relationship between a rectangular prism and a triangular prism. Because of the time spent on activating their prior knowledge, the students are able to see the relationship that exists between a rectangular and a triangular base. I have students work through a process in small groups to discuss possible relationships between rectangular prisms of various sizes, and then we look at what the volume of a prism measuring 1x3x4 would be. We then keep adding on another 3x4 and another 3x4 and students begin to notice the relationship. (See "Volume of Prisms and Pyramids" activity in the linked folder)

Figure 9

In seventh grade math students are also introduced to pyramids. Once they understand prisms, we then look at the relationship between the volume of a rectangular prism and the volume of a rectangular pyramid. Using the solids of a square pyramid and a cube, with each having the same size base and height, students measure to see how much fluid volume they can get into that cube using the square pyramid as the measuring tool. I warn you this can be messy, but it is fun and they never forget the exploration!

(There is also a YouTube video of the same process you can show without all the mess. It can be found at: https://www.youtube.com/watch?v=OUDjY6vJ8pw&list=PLBPsepo80Eiw0hZ-LAL57YJiMlYvl0YNH&index=6) From there we have discussions about the relationship of squares and triangles and how that connection is present in calculating volume of a square pyramid and the volume of a triangular pyramid. The result is amazing.

I believe exploration is an important part of math instruction, and concrete manipulatives are tools that allow us to provide students with opportunities to make connections and think deeply about mathematics. These connections create problem solvers. Along with the use of manipulatives, the regular use of rich tasks having multiple entry points is an instrumental part of good math instruction. I came across this quote: “Rich tasks allow the learner to 'get inside' the mathematics. The resulting learning process is far more interesting, engaging and powerful; it is also far more likely to lead to a lasting assimilation of the material for use in both further mathematical study and the wider context of applications.”(Hewson, 2011) I wholeheartedly believe in this statement.

I want to end with one of my favorite quotes from Marilyn Burns:

> “Without manipulatives, children are too often lost in a world of abstract symbols for which they have no concrete connection or comprehension.”

References

“Algebra Tiles.ppt - Let\u2019s Do Algebra Tiles David McReynolds AIMS PreK-16 Project Noel Villarreal South Texas Rural Systemic Initiative Algebra Tiles: Course Hero.” *Algebra Tiles.ppt - Let\u2019s Do Algebra Tiles David McReynolds AIMS PreK-16 Project Noel Villarreal South Texas Rural Systemic Initiative Algebra Tiles | Course Hero*, 12 Jan. 2021, www.coursehero.com/file/77757211/Algebra-Tilesppt/.

Burns, M. (n.d.). How to make the most of manipulatives. Found August 28, 2016 at: http://teacher.scholastic.com/lessonrepro/lessonplans/instructor/burns.htm

Cuisenaire Rods: the Super Source: Grades 3-4. Cuisenaire Company of America, 1996.

Furner, Joseph M. and Worrell, Nancy L. (2017) "The Importance of Using Manipulatives in Teaching Math Today," Transformations: Vol. 3 : Iss. 1 , Article 2.

https://studylib.net/doc/10257110/welcome-to--using-manipulatives-in-the-secondary

“Implement Tasks That Promote Reasoning and Problem Solving." Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM, National Council of Teachers of Mathematics, 2014. 17-24. Print.

Laura O'Connor, and Debra Stoll. AngLegs Activity Cards. Learning Resources, 2007.

Welchman-Tischler, Rosamond. Teaching with Manipulatives: Middle School Investigations. ETA/Cuisenaire, 2003.

“What Is a Mathematically Rich Task?” *NRICH*, nrich.maths.org/6299.