WISCONSIN MIDDLE SCHOOL STATE MATHEMATICS MEET
WISCONSIN MATHEMATICS COUNCIL
March 6 – 10, 2017

Solutions

Problem set #1

1. 4 🍎 5 is 4 + 4 – 5 = 3, so 6 🍎 □ = 3. 6 + 6 – □ = 3 → 12 – □ = 3 → □ = **9.**

2. There are 21 triangles of size 1 that point up. There are 15 triangles of size 4 (look at the top two rows of the figure) that point up. There are 10 triangles of size 9, 6 triangles of size 16, 3 triangls of size 25, and 1 triangle of size 36 (the entire figure). 21 + 15 + 10 + 6 + 3 + 1 = **56** triangles.

   Extra credit: Using a similar method, can you count the number of triangles that point down?

3. It is a known fact about parallelograms (and all subclasses of parallelograms) that opposite sides have the same length. First, look at the lengths of *MH* and *AT*. In an equation, set them equal to each other: $108 = 15x + 3$. This solves as $x = 7$. Use this in another equation, setting *MA* and *TH* equal to each other: $3xy^3 = 168 \rightarrow 3(7)y^3 = 168 \rightarrow 21y^3 = 168 \rightarrow y^3 = 8$, so $y = 2$. If $x = 7$ and $y = 2$, then $y^x = 2^7 =$ **128.**

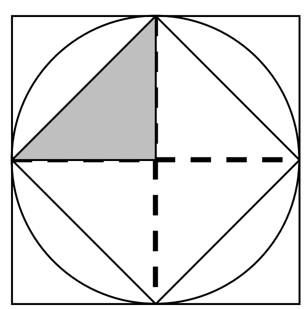
Problem set #2

1. Look at the first and third equation, and ignore the second one. Replace the triangle in the first equation with the two squares from the third equation, and then you have one square equalling **2 circles.**

2. In one hour, Tom will paint $\frac{1}{8}$ of the fence. In the same hour, Sara will paint $\frac{1}{4}$ of the fence. Together, they have painted $\frac{3}{8}$ of the fence in one hour. $\frac{\frac{3}{8}\text{ fence painted}}{1\text{ hour}} = \frac{1\text{ fence painted}}{x\text{ hours}}$.

   Solving this equation by cross multiplying gives us $x = \frac{8}{3}$ hours = **2 hours 40 minutes.**

3. Rotate the central square 45°, and ignore the circle. You will notice that if you divide the figure as shown with dashed lines, the shaded part in the upper left quadrant is half of the upper left quadrant. This makes the central square half as big as the outer square, so the ratio of the areas is **1 : 2.**



Problem set #3

1. The original area is $4 \cdot 6 = 24 \text{ in}^2$. After being enlarged, it is now $8 \cdot 10 = 80 \text{ in}^2$. The percent increase in area is $\frac{80-24}{24} = \frac{56}{24} = 2.\bar{3} = \boxed{\textbf{233\%.}}$

2. Let n be the number of games bowled before Mary's average became 198. Then to find n, $\frac{164n+198}{n+1} = 165$, so $164n + 198 = 165n + 165$, or $n = 33$. Her 34th game raised her average from 164 to 165. To raise her average to 166 with her 35th game, we add one more bowling game into the average calculation: $\frac{164(33)+198+x}{35} = 166$, which becomes $5610 + x = 5810$, or $x = \boxed{\textbf{200.}}$

3. If the three numbers are each divisible by 6, then they are each also divisible by 2. To be divisible by 2, each number must be even, so each of A, B, and C must be even numbers. To be divisible by 9, add the three digits, and the sum must be divisible by 9. If all the digits are even, then that sum must be even, therefore the sum of the digits cannot be 9 or 27, it must be 18. Finally, if the digits are all distinct, then they must be 4, 6, and 8.

   In no particular order, we have $468 + 684 + 846 = \boxed{\textbf{1,998.}}$

   Extra credit: How does this problem change if A, B, and C are not necessarily distinct?

Problem set #4

1. $A = \{5, 6, 7, 8, 9\}$ (can be found algebraically or through guess and check). $B = \{7, 8, 9, 10\}$. The *intersection* of these two sets is the set of elements that are common to both sets. In this case, the intersection of A and B is $\{7, 8, 9\}$. The intersection contains $\boxed{\textbf{3}}$ integers. Using set symbol notation, we say that $A \cap B = \{7, 8, 9\}$. $|A \cap B| = 3$.

2. Since it does not matter whether the existing children are boys or girls, that information has no bearing on whether future children will be boys or girls, so we can ignore the fact that the first three children were all boys. Looking to the future, the next two children will be one of these four outcomes: {BB, BG, GB, GG}. Three of these outcomes contain at least one girl, so the probability that the couple will have at least one girl is $\boxed{\mathbf{\frac{3}{4}.}}$

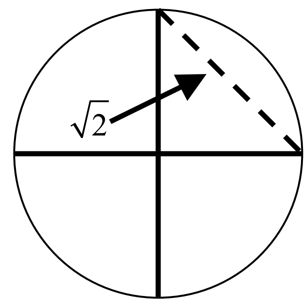
3. Let x and y be the two numbers. Note that it is not necessary for us to find their values.

   $xy = 20, (x-1)(y-1) = 13 \rightarrow xy - x - y + 1 = 13$. Substituting 20 into the last equation, we get $20 - x - y + 1 = 13 \rightarrow \boxed{\textbf{8}} = x + y$.

Team Problem set

1. Each vertex will be added to label the three edges they connect to, and each edge will be added to label both faces they are connected to. As a result, each vertex gets added a total of 6 times. Add the values of all six vertices to get 16, and $6 \cdot 16 =$ $\boxed{\textbf{96.}}$

2. Look at the picture to the right, of a circle divided into four quadrants. The question asks to pick 5 points randomly positioned within the circle, and see if there is a way to have at least two of them be a distance of 1.5 units away from each other. Using the Pigeonhole Principle, let's put one point into each of the four quadrants (or on a boundary between two quadrants). The fifth point has to go into one of the quadrants that already has a point in it. The farthest those two points can be from each other is $\sqrt{2} \approx 1.414$ units apart. And if you try to space any of the other points away from each other, you can get them to move away from a point on one side of the quadrant, but there will be another quadrant nearby where it is trying to do the same thing but can't because it will have to be near the first point. Repeated trial and error will reveal that there is no way to space these points so that you can make every group of two out of the five of them 1.5 units apart, so the probability we are looking for is $\boxed{\textbf{1.}}$



3. A mean of 6 means that the three numbers add up to 18. A median of 7 and no mode means that one of the numbers is smaller than 7, while the other number is larger than 7. If 7 is one of the numbers, then the other two numbers have to add up to 11. The only pairs of numbers that will work are (1, 10), (2, 9), and (3, 8). Therefore, the three distinct sets of numbers meeting these criteria are $\boxed{\textbf{\{1, 7, 10\}, \{2, 7, 9\}, and \{3, 7, 8\}.}}$

4. Let x be the number of 35 cent stamps, $3x$ be the number of 20 cent stamps, and y be the number of 50 cent stamps. Gonzo bought $4x + y$ stamps in all. Write this equation:

$$35x + 20(3x) + 50y = 2000$$
$$95x + 50y = 2000$$
$$19x + 10y = 400$$

Rewrite the last equation as $19x = 400 - 10y = 10(40 - y)$. This means that x is a multiple of 10, and $40 - y$ is a multiple of 19. If $x = 10$ and $y = 21$, then Gonzo bought 61 stamps. Instead, if $x = 20$ and $y = 2$, then Gonzo bought 82 stamps. The problem states that he bought fewer than 80 stamps, so this second solution is incorrect. Gonzo bought $\boxed{\textbf{61 stamps.}}$

5. To maximize the number of dartboard misses, you need to minimize the number of darts that hit the scoring regions. There are three ways to do this. If the total score is 37 points, then you could have gotten seven 5s and two 1s, allowing you to miss the board just once. Also, you could get six 5s, two 3s, and one 1, again allowing you to miss the board just once. Or you can get five 5s and four 3s, missing once. Therefore, the most number of misses is $\boxed{\textbf{1.}}$

6. Label the top number A, the two across the middle row B on the left and C on the right, and the bottom three from left to right D, E, and F. $A+B+D=A+C+F=D+E+F=S$. We also know that $A+B+C+D+E+F=135$. Let's add all three sides together:

$$\begin{aligned}(A+B+D)+(A+C+F)+(D+E+F)&=3S\\ 2A+B+C+2D+E+2F&=3S\\ 135+A+D+F&=3S\end{aligned}$$

To get the largest possible value for S, we need the largest possible value for $A+D+F$ that is divisible by 3 (both 135 and $3S$ are divisible by 3, so $A+D+F$ must be too). Let A = 25, D = 24, and F = 23 (without loss of generality; these three numbers can be put in any corner positions and it still works). 20 fits between 24 and 25, 21 fits between 23 and 25, and 22 fits between 23 and 24. This makes the largest (and only) value of $S=$ **69.**

Extra Credit

Problem Set #1, 2: There are 15 triangles of size 1, 6 triangles of size 4, and 1 triangle of size 9, for a total of **22** triangles that point down. There are vastly fewer of these triangles, since the bases of the triangles cannot be longer than the width of the triangle, so that limits the largest of these triangles to being one-fourth of the total size of the largest triangle.

Problem Set #3, 3: There is no change to the answer if the digits are not necessarily distinct. They still have to add up to 18, and they all must still be even. So in addition to (A, B, C) being (4, 6, 8), they could also be (2, 8, 8) or (6, 6, 6). The results are the same:

$$288+828+882=1{,}998$$
$$666+666+666=1{,}998$$