

PROBLEM-SOLVING SITUATIONS

| JOINING PROBLEMS | | |
|---|--|---|
| Join: Result Unknown (JRU) | Join: Change Unknown (JCU) | Join: Start Unknown (JSU) |
| <p>♦ Grandmother had 5 strawberries. Grandfather gave her 8 more strawberries. How many strawberries does Grandmother have now?</p> <p align="center">$5 + 8 = \square$</p> | <p>♥ Grandmother had 5 strawberries. Grandfather gave her some more. Then Grandmother had 13 strawberries. How many strawberries did Grandfather give Grandmother?</p> <p align="center">$5 + \square = 13$</p> | <p>♠ Grandmother had some strawberries, Grandfather gave her 8 more. Then she had 13 strawberries. How many strawberries did Grandmother have before Grandfather gave her any?</p> <p align="center">$\square + 8 = 13$</p> |
| SEPARATING PROBLEMS | | |
| Separate: Result Unknown (SRU) | Separate: Change Unknown (SCU) | Separate: Start Unknown (SSU) |
| <p>♦ Grandfather had 13 strawberries. He gave 5 strawberries to Grandmother. How many strawberries does Grandfather have left?</p> <p align="center">$13 - 5 = \square$</p> | <p>♥ Grandfather had 13 strawberries. He gave some to Grandmother. Now he has 5 strawberries left. How many strawberries did Grandfather give Grandmother?</p> <p align="center">$13 - \square = 5$</p> | <p>♠ Grandfather had some strawberries. He gave 5 to Grandmother. Now he has 8 strawberries left. How many strawberries did Grandfather have before he gave any to Grandmother?</p> <p align="center">$\square - 5 = 8$</p> |
| PART-PART-WHOLE PROBLEMS | | |
| Part-Part-Whole: Whole Unknown (PPW:WU) | Part-Part-Whole: Part Unknown (PPW:PU) | |
| <p>♦ Grandmother has 5 big strawberries and 8 small strawberries. How many strawberries does Grandmother have altogether?</p> <p align="center">$5 + 8 = \square$</p> | <p>♥ Grandmother has 13 strawberries. Five are big and the rest are small. How many small strawberries does Grandmother have?</p> <p align="center">$13 - 5 = \square$ or $5 + \square = 13$</p> | |
| COMPARE PROBLEMS | | |
| Comp. Difference Unknown | Comp. Quantity Unknown | Comp. Referent Unknown |
| <p>♦ ♥ Grandfather has 8 strawberries. Grandmother has 5 strawberries. How many more berries does Grandfather have than Grandmother?</p> <p align="center">$8 - 5 = \square$ or $5 + \square = 8$</p> | <p>♠ Grandmother has 5 strawberries. Grandfather has 3 more strawberries than Grandmother. How many strawberries does Grandfather have?</p> <p align="center">$5 + 3 = \square$</p> | <p>♠ Grandfather has 8 strawberries. He has 3 more strawberries than Grandmother. How many strawberries does Grandmother have?</p> <p align="center">$8 - 3 = \square$ or $\square + 3 = 8$</p> |
| MULTIPLICATION & DIVISION PROBLEMS | | |
| Multiplication | Measurement Division | Partitive Division |
| <p>♦ Grandmother has 4 piles of strawberries. There are 3 strawberries in each pile. How many strawberries does Grandmother have?</p> <p align="center">$4 \times 3 = \square$</p> | <p>♦ Grandmother had 12 strawberries. She gave them to some children. She gave each child 3 strawberries. How many children were given strawberries?</p> <p align="center">$12 \div 3 = \square$</p> | <p>♦ ♥ Grandfather has 12 strawberries. He wants to give them to 3 children. If he gives the same number of strawberries to each child, how many strawberries will each child get?</p> <p align="center">$12 \div 3 = \square$</p> |

Problem chart based on Cognitively Guided Instruction Problem Types (Carpenter et al., 1996)



MATH STORY PROBLEM TYPES

JOINING PROBLEMS

| Join (Result Unknown) $6 + 3 = \underline{\quad}$ | Join (Change Unknown) $4 + \underline{\quad} = 7$ | Join (Start Unknown) $\underline{\quad} + 4 = 6$ |
|--|--|--|
| Mr. Smith had 6 cookies. Suzy gave him 3 more cookies. How many cookies does Mr. Smith have now? | Mr. Smith had 4 cookies. Suzy gave him some more. Then, Mr. Smith had 7 cookies. How many cookies did Suzy give Mr. Smith? | Mr. Smith had some cookies. Suzy gave him 4 more cookies. Then, he had 6 cookies. How many cookies did Mr. Smith start with? |

SEPARATING PROBLEMS

| Separate (Result Unknown) $7 - 4 = \underline{\quad}$ | Separate (Change Unknown) $5 - \underline{\quad} = 1$ | Separate (Start Unknown) $\underline{\quad} - 4 = 4$ |
|---|---|--|
| Mr. Smith had 7 cookies. He gave 4 of them to Suzy. How many cookies did Mr. Smith have left? | Mr. Smith had 5 cookies. He gave some to Suzy. Then, he had 1 cookie left. How many cookies did Mr. Smith give to Suzy? | Mr. Smith had some cookies. He gave 4 to Suzy. Then, he had 4 cookies left. How many cookies did Mr. Smith have to start with? |

PART - PART - WHOLE PROBLEMS

| Part - Part - Whole (Whole Unknown) $6 + 3 = \underline{\quad}$ | Part - Part - Whole (Part Unknown) $7 - 4 = \underline{\quad}$ or $4 + \underline{\quad} = 7$ |
|---|--|
| Mr. Smith had 6 white cookies and 3 pink cookies. How many cookies did Mr. Smith have altogether? | Mr. Smith had 7 cookies. 4 were pink and the rest were white. How many white cookies did Mr. Smith have? |

COMPARING PROBLEMS

| Compare (Difference Unknown) $5 - 3 = \underline{\quad}$ or $3 + \underline{\quad} = 5$ | Compare (Quantity Unknown) $3 + 2 = \underline{\quad}$ | Compare (Referent Unknown) $8 - 5 = \underline{\quad}$ |
|--|--|---|
| Mr. Smith had 5 cookies. Suzy had 3 cookies. How many more cookies did Mr. Smith have than Suzy? | Mr. Smith had 3 cookies. Suzy had 2 more cookies than Mr. Smith. How many cookies did Suzy have? | Mr. Smith had 8 cookies. He had 5 more than Suzy. How many cookies did Suzy have? |

MULTIPLYING AND DIVIDING PROBLEMS

| Multiplication $3 \times 3 = \underline{\quad}$ | Measurement Division $9 \div 3 = \underline{\quad}$ | Partitive Division $12 \div 3 = \underline{\quad}$ |
|---|--|--|
| Mr. Smith had 3 piles of cookies. There were 3 cookies in each pile. How many cookies did Mr. Smith have? | Mr. Smith had 9 cookies. He put 3 cookies in each box. How many boxes did he need? | Mr. Smith had 12 cookies. He wanted to give them to 3 friends. How many cookies did each friend get? |

CGI Problem Types

| | | | |
|-----------------|--------------------|--------------------------|----------------------|
| JOIN | Result Unknown | Change Unknown | Start Unknown |
| SEPARATE | Result Unknown | Change Unknown | Start Unknown |
| Part-Part-Whole | Whole Unknown | Part Unknown | |
| Compare | Difference Unknown | Compare Quantity Unknown | Referent Set Unknown |

| | | |
|----------------|----------------------|--------------------|
| Multiplication | Measurement Division | Partitive Division |
|----------------|----------------------|--------------------|

When planning the next level of challenge for each student, teachers use the *MMSD K-5 Grade Level Mathematics Standards* to guide the selection of problem types and number sizes. Teachers know that every student will take a unique path in becoming proficient with the problem types, solution strategies and number sizes. An overview of the K-2 development as described in the *MMSD standards* appears in the following table:

| Kindergarten | First Grade | Second Grade |
|--|--|---|
| <p>Problem types:</p> <ul style="list-style-type: none"> Join, Result Unknown Separate, Result Unknown Multiplication Measurement Division Partitive Division, sharing by 2 <p>Solution strategies:</p> <ul style="list-style-type: none"> Modeling strategies Acting it out Using objects Drawing pictures <p>Counting strategies:</p> <ul style="list-style-type: none"> Moving into counting on 1 and 2 <p>Number Sizes:</p> <ul style="list-style-type: none"> Modeling strategies: 0 – 20; focus on 0 – 10 | <p>Problem types:</p> <ul style="list-style-type: none"> Join and Separate, Result Unknown Multiplication Measurement Division Partitive Division, sharing by 2, 3 and 4 Join and Separate, Change Unknown Compare, Difference Unknown Part, Part, Whole, Whole Unknown <p>Solution strategies:</p> <ul style="list-style-type: none"> Counting strategies Counting on or back 1, 2, 3 Counting by groups of 2, 5, 10 <p>Modeling strategies:</p> <ul style="list-style-type: none"> Acting it out Using objects Drawing pictures Using empty number lines <p>Number sizes:</p> <ul style="list-style-type: none"> Mental computations: Addition – 0 – 10, moving into 0 – 20 Modeling strategies: 0 – 100 | <p>Problem type:</p> <ul style="list-style-type: none"> Join and Separate, Result Unknown Multiplication Measurement Division Partitive Division, sharing by 2, 3, 4, 6 and 8 Join and Separate, Change Unknown Compare, Difference Unknown Part, Part, Whole, Whole Unknown Part, Part, Whole, Part Unknown Join and Separate, Start Unknown <p>Solution strategies:</p> <ul style="list-style-type: none"> Counting strategies Counting on or back 1, 2, 3 Counting by groups of 2, 5, 10 Decomposing numbers Using landmarks (10) Using place value concepts <p>Modeling strategies:</p> <ul style="list-style-type: none"> Acting it out Drawing pictures Using objects Using empty number lines Using arrow language <p>Number sizes:</p> <ul style="list-style-type: none"> Mental computations: Addition – Sums from 0 – 20 Place value concepts – 0 – 100 Subtraction – Differences of 1, 2 or 3 Modeling and counting strategies: Numbers beyond 100 |

Name/Grade

Green Type CGI Problems

| Clip Number | Clip 4.1 | Clip 3.16 | Clip 3.2 | Clip 4.5 | Clip _____ | Clip _____ | Clip _____ |
|---------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Problem Type | JRU | JRU | JRU | JRU | JRU | JRU | JRU |
| | SRU | SRU | SRU | SRU | SRU | SRU | SRU |
| | Multiplication | Multiplication | Multiplication | Multiplication | Multiplication | Multiplication | Multiplication |
| | PPW:WU | PPW:WU | PPW:WU | PPW:WU | PPW:WU | PPW:WU | PPW:WU |
| Strategy Used | No Strategy | No Strategy | No Strategy | No Strategy | No Strategy | No Strategy | No Strategy |
| | Direct Modeling | Direct Modeling | Direct Modeling | Direct Modeling | Direct Modeling | Direct Modeling | Direct Modeling |
| | Counting Strategy | Counting Strategy | Counting Strategy | Counting Strategy | Counting Strategy | Counting Strategy | Counting Strategy |
| | Derived | Derived | Derived | Derived | Derived | Derived | Derived |
| Notes | | | | | | | |

| | JRU | SRU | M | PPW:WU |
|----------------|--|--|---|--|
| Strategy Used: | No Strategy | No Strategy | No Strategy | No Strategy |
| | <p>Direct Modeling- Using a manipulative, student will make a pile of 5 and a pile of 3, will join them all together and then count each one.</p> <p>Counted On- Says 5 or 3, then will count on their fingers the rest of the way. Ex: 5.....6, 7, 8 or 3... 4, 5, 6, 7, 8</p> <p>Derived- Knew that 5 + 3 is 8 because they are fluent. When using 15, 13 the student might say "10 + 10 is 20, and 5 + 3 is 8, so 20 and 8 is 28." Able to solve mentally with a strategy.</p> | <p>Direct Modeling- Using a manipulative, student will make a pile of 8 and take away 3, then count each one to find the answer.</p> <p>Counted back/Added On- Counts back from 8 three times to get 5, or counts back until they get to 3 to get 5.</p> <p>Derived- Able to solve mentally. When using bigger numbers, students are able to break apart (decompose) numbers to make the problem easier for them. Ex: "18-10 is 8, then take 3 more away to get 5."</p> | <p>Direct Modeling- Using manipulatives, student will create an area for 3 baskets, and put two cubes in each basket. Will then count each cube.</p> <p>Skip Counting- Using manipulatives or fingers, student will count by 2's to solve the problem.</p> <p>Derived- Able to solve mentally. When using bigger numbers, student will be able to break apart (decompose) numbers to make the problem easier for them. Ex: "5x5 is 25, plus 5 more is 30."</p> | <p>Direct Modeling-Using a manipulative, student will make a pile of 5 and a pile of 4, will join them all together and then count each one.</p> <p>Counted On- Says 5 or 4, then will count on their fingers the rest of the way. Ex: 5.....6, 7, 8, 9 or 4...5, 6, 7, 8, 9</p> <p>Derived-Knew that 5 + 4 is 8 because they are fluent. When using 15, 14 the student might say "10 + 10 is 20, and 5 + 4 is 9, so 20 and 9 is 29." Able to solve mentally with a strategy.</p> |