Authors: David Berger, Diana Braham, Adam Chawansky, Cesar Ebonia, Jake Leibold, & Mary E. Pilgrim.
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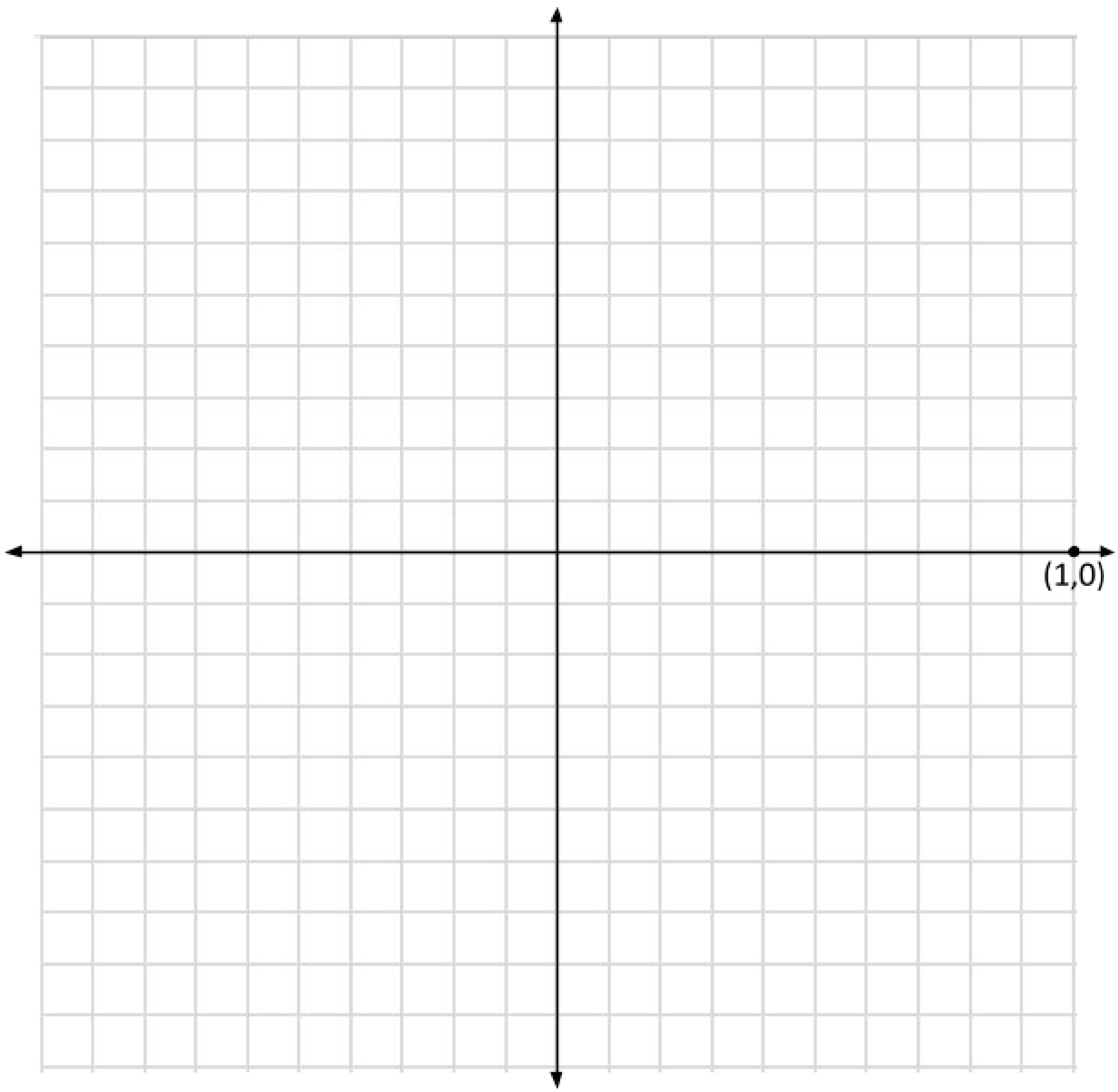
Name ____ Date ____________________

Trigonometric Exploration!

Use your triangles to plot points on the coordinate plane below.

1) Place the labeled angle at the origin and the darkened base along the x-axis.
2) Plot the point at the vertex of the other acute angle.
3) Label the coordinates of the point.



<u>After</u> you have plotted your points:

- Glue <u>four copies of one of the triangles</u> in each of the four quadrants (again, making sure the labeled angle is at the origin and the darkened base is along the x-axis). Then, glue your other three triangles here!

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Important Questions

1) What seems to be true about all the points that you plotted?

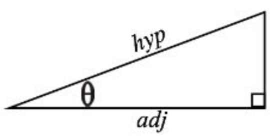
2) How could you find more points that fit the pattern you described above?

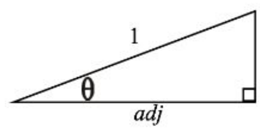
3) Why should the points $(1,0)$, $(0,1)$, $(-1,0)$, and $(0,-1)$ be included?

4) How do you find the length of *adj*…

a) in terms of θ and *hyp?*

b) what if *hyp* = 1?

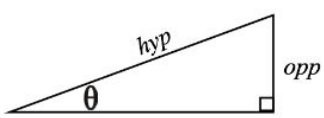


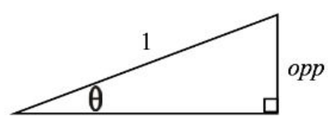


5) How do you find the length of *opp*…

a) in terms of θ and *hyp?*

b) what if *hyp* = 1?





6) Looking at the triangles you glued on the graph, which coordinate is the *adjacent* side? which is the *opposite* side?

adjacent -->

opposite -->

Combining your work from questions 4b), 5b), and 6), write the ordered pair for the yellow triangle in the first quadrant (like you did for all of the other triangles).

7) So, in the unit circle,

$\cos\theta$ = __________

$\sin\theta$ = __________

8) What is the equation of your circle? (Hint: Pythagorean Theorem?)

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Radian Round-Up

9) What does a radian measure? How would you define it?

10) What is the connection between radians and degrees?

11) Make a conjecture: How do you convert between radians and degrees? Hint: What do you know about degree measurements and radian measurements of a circle?

12) Label the radian measure for each of the points you plotted on your unit circle.

A-Ha!

13) So now can you find…

$sin\ \frac{\pi}{4} =$ $\quad$ $cos\ \frac{\pi}{3} =$ $\quad$ $sin\ \frac{5\pi}{4} =$ $\quad$ $cos\ \pi =$ $\quad$ $sin\ \frac{3\pi}{2} =$

Tough Stuff

14) What is…

$sin\ \frac{23\pi}{6} =$ $\quad$ $cos\ \frac{21\pi}{4} =$ $\quad$ $cos\ (-\frac{\pi}{3}) =$

15) How many solutions are there to the equation $sin\ \theta = \frac{1}{2}$?

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Other Considerations...

16) What does tangent "mean" in the context of the unit circle? (Use ideas from Problem 6.)

17) The values of cosine and sine repeat after every 2π. How often do the values of tangent repeat? Explain why using the unit circle.

18) What relationships can you find between $\cos\theta$ and $\sin\theta$ by investigating the unit circle?

19) The half angle identity—one trigonometric relationship—is written below. Can you prove the relationship is true for all θ using the unit circle?

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

20) Find an exact value using radicals for other values of sine.

$$\sin\frac{\pi}{12} = \qquad\qquad \sin\frac{\pi}{10} =$$

21) Eric used calculus to derive polynomial approximations to cosine and sine. What connections can you find between the polynomial expansions and the unit circle?

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\ldots \qquad\qquad \cos x \approx 1 - x^2 + \frac{x^4}{4!} - \frac{x^6}{6!}\ldots$$

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Supplemental Think-Pair-Share Questions

1. **Why is the circle you created called the unit circle?**

2. **Where can you look on the unit circle to find the value of cos(30°) or cos(60°)? Why?**

3. **Where can you look on the unit circle to find the value of sin(30°) or sin(45°)?**

4. **Where would you look to find the sin(135°)?**

5. **Where would you look to find the cos(-60°)?**

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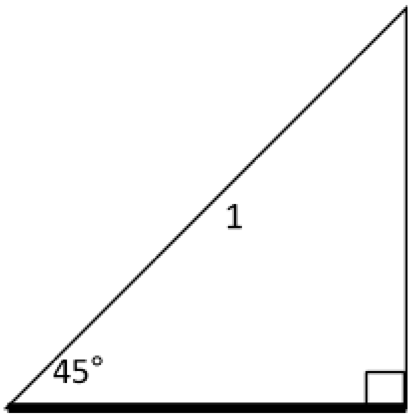
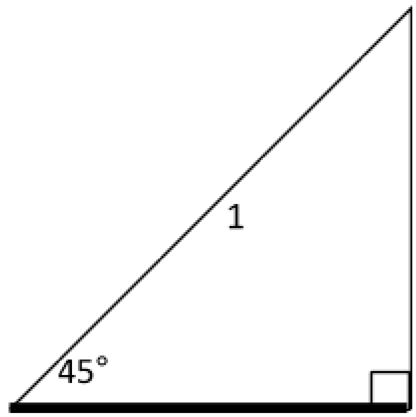
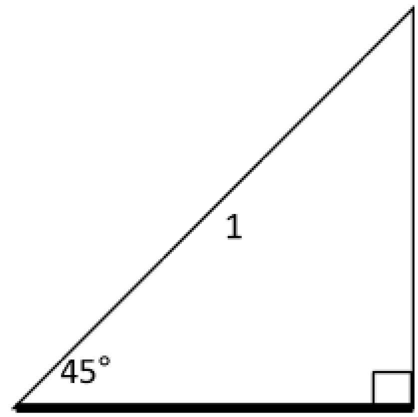
45-45-90







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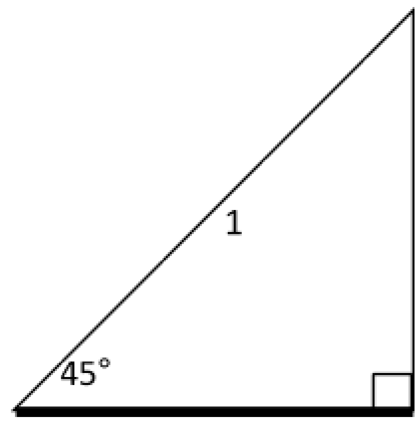

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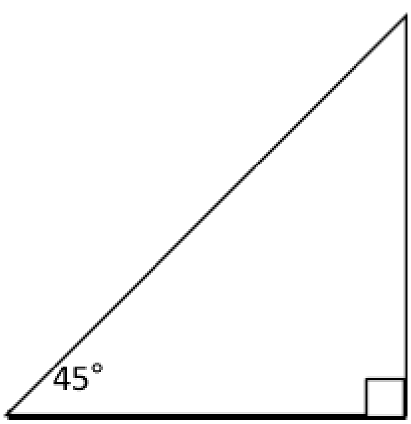

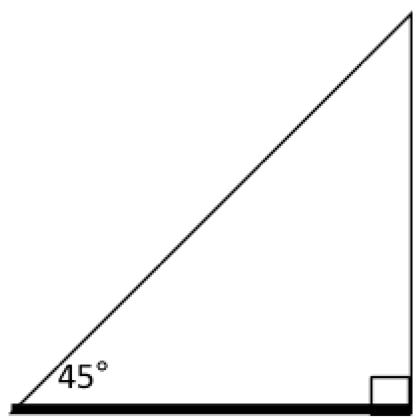

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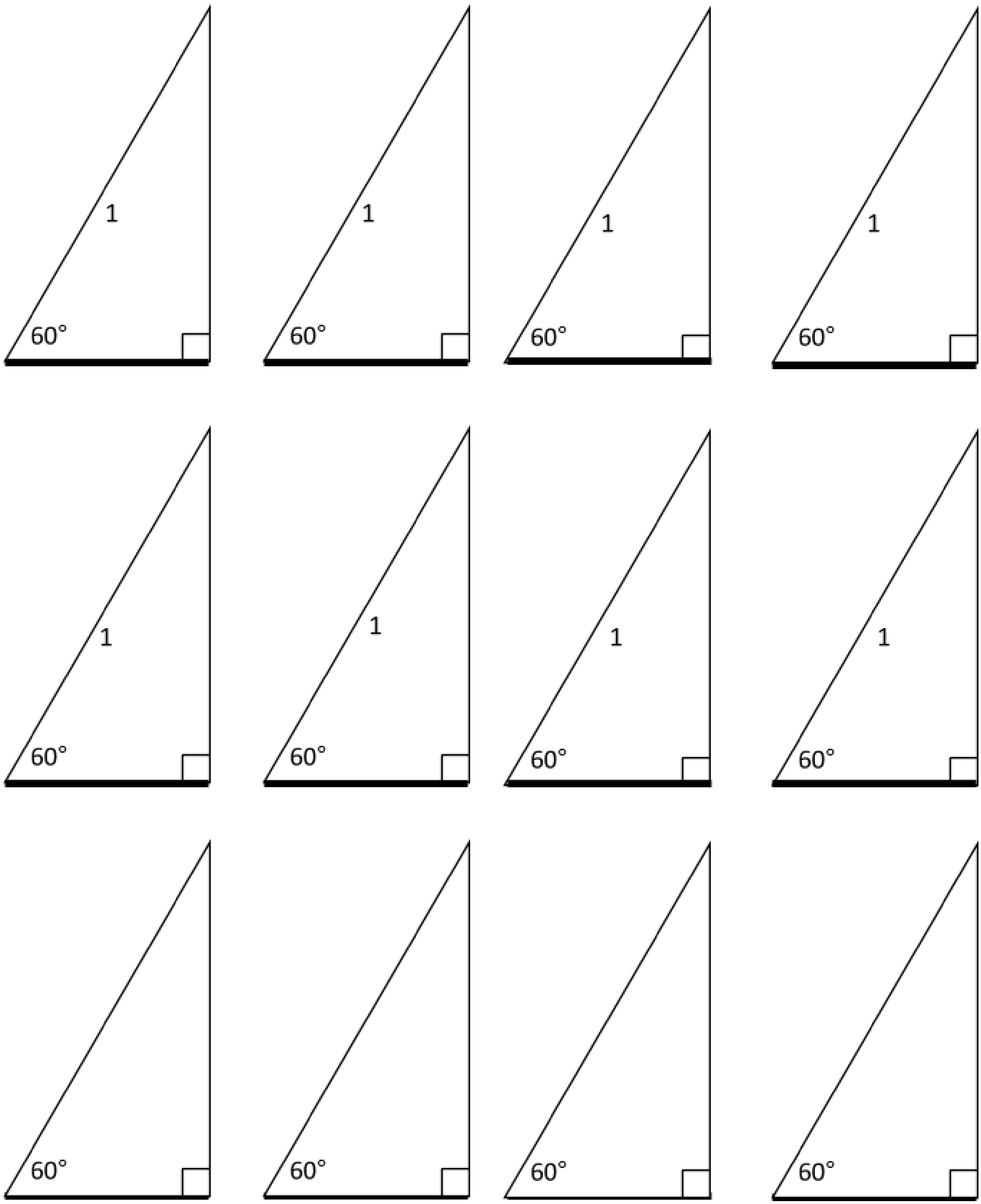
30-60-90



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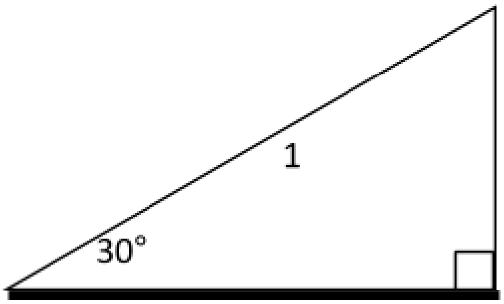
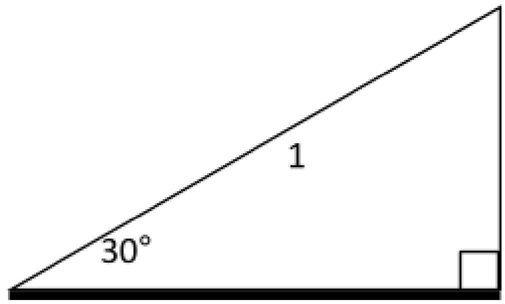
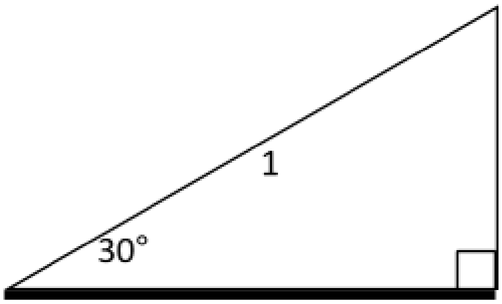
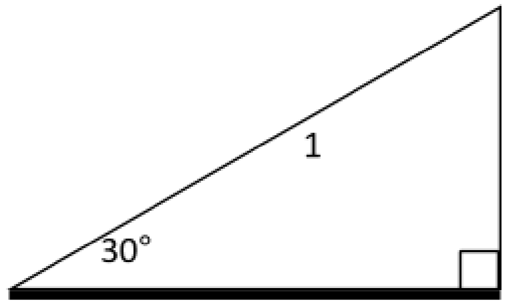
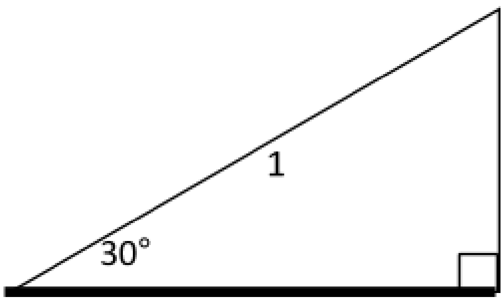
30-60-90

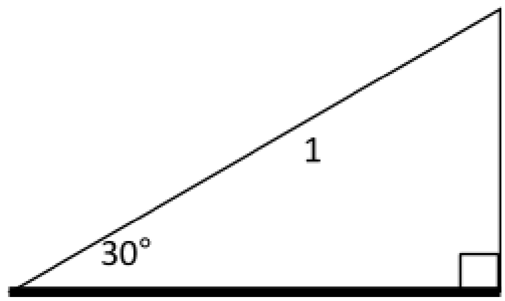


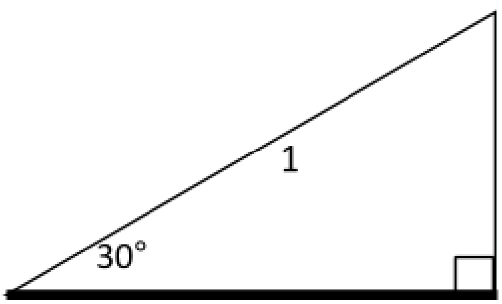


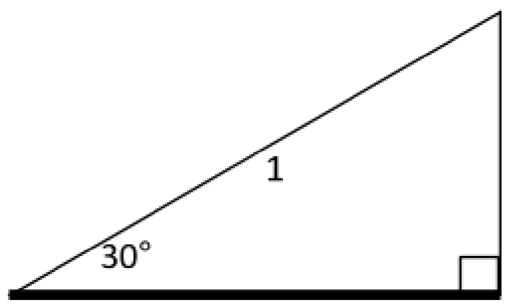


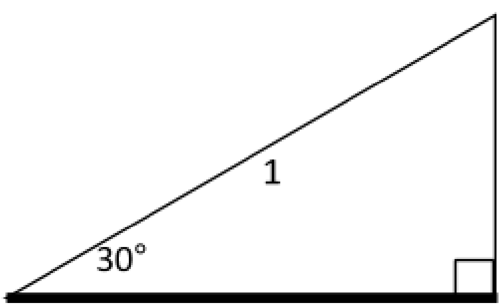


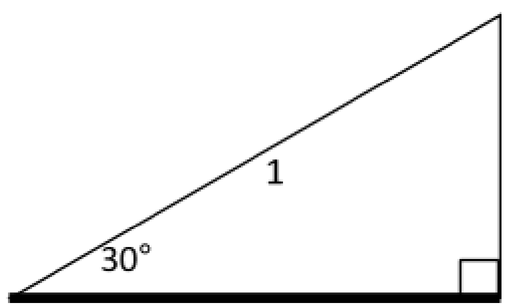












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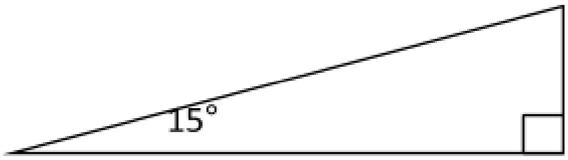
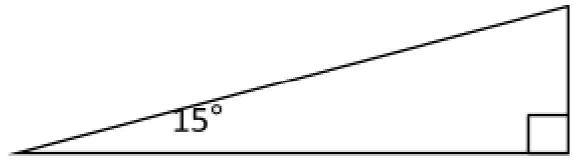
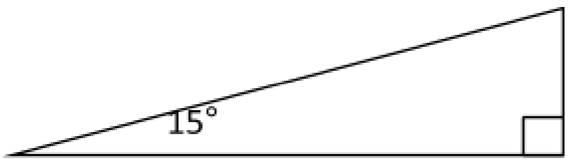
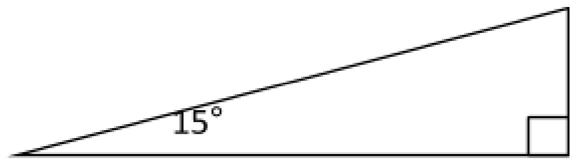
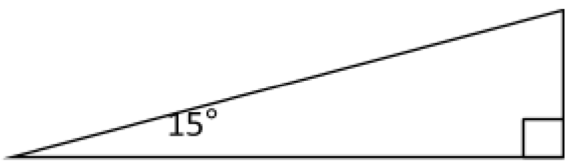
15-75-90

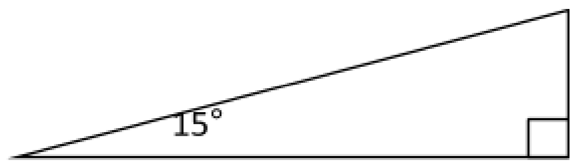


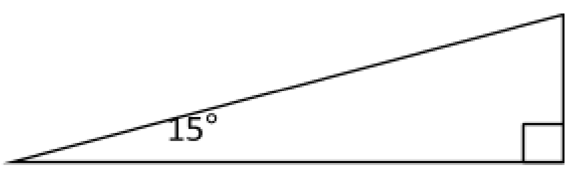


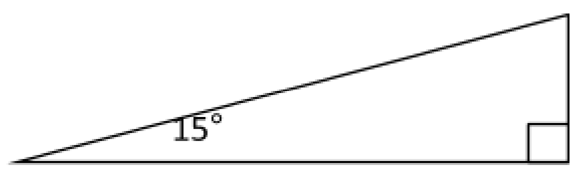


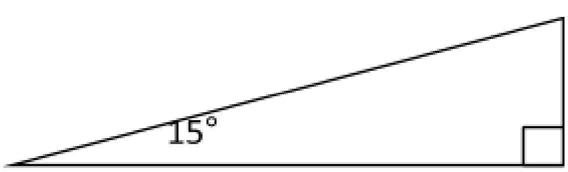


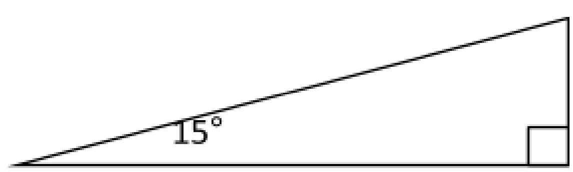


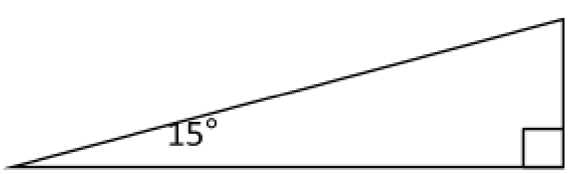


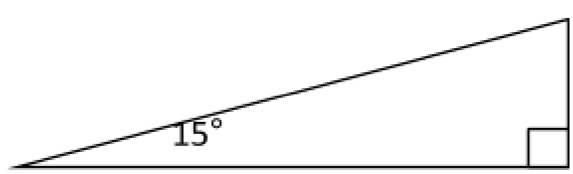












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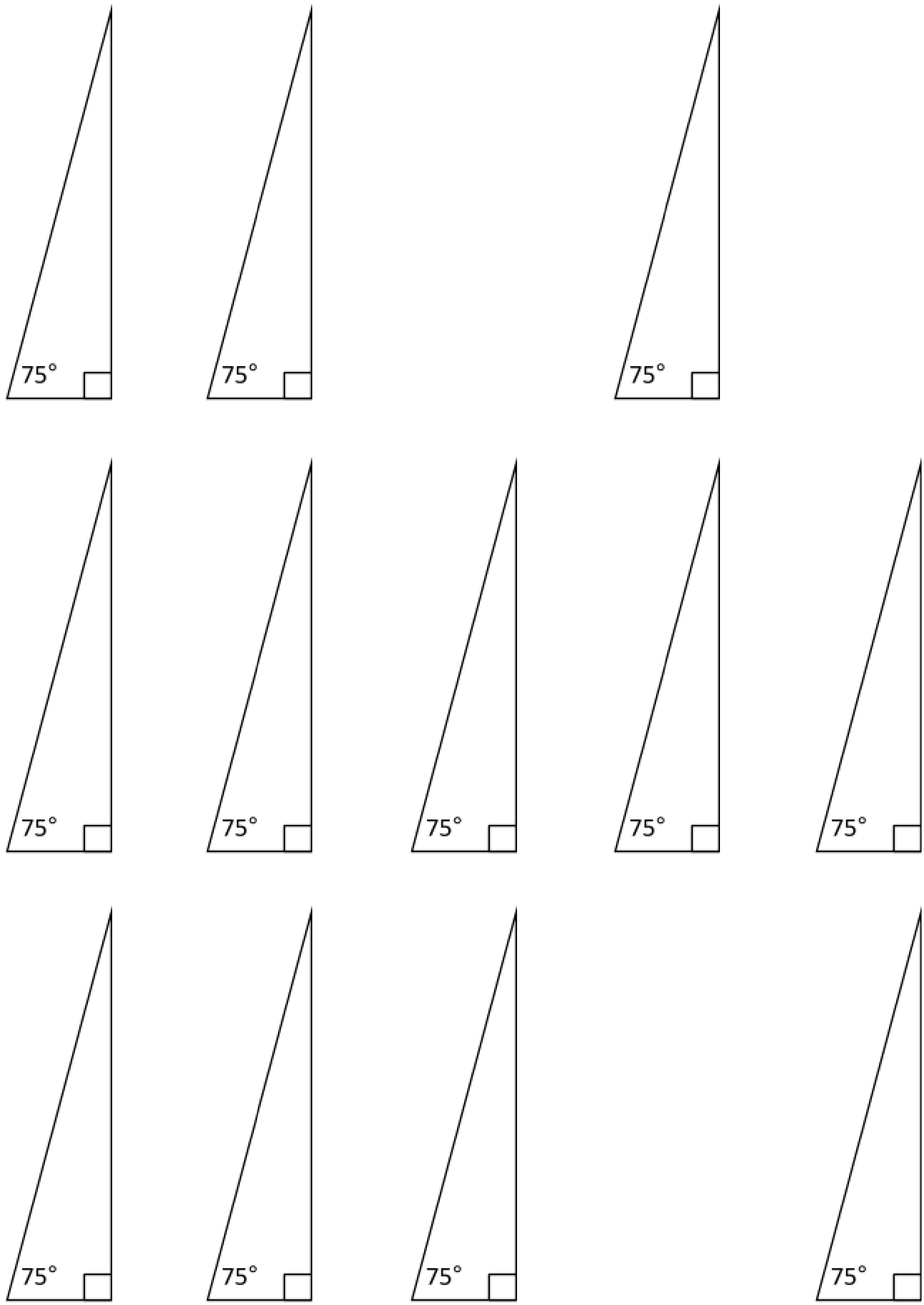

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70-20-90